Application Note: Estimating the Current Measurement Uncertainty when utilizing Current Transducers

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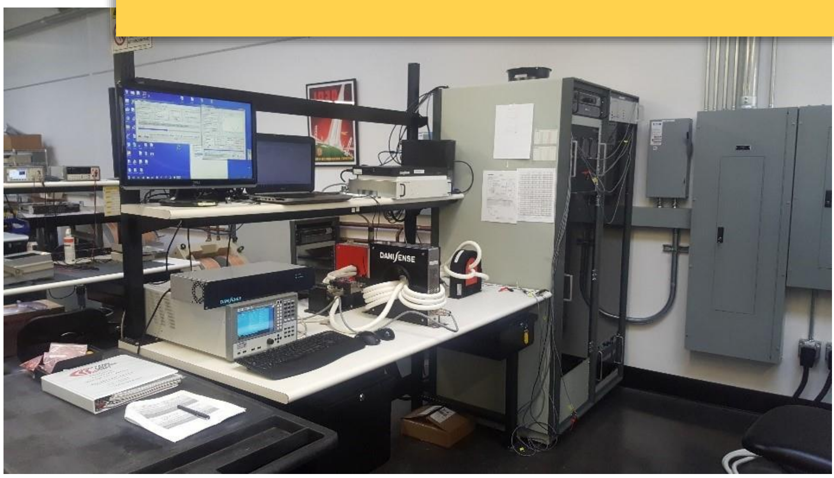

Overview

When performing current measurements using a Current Transducer, the following question often comes up: *What is the Measurement Uncertainty?* If we were directly measuring the current, say with a DMM in series, then the answer is relatively straightforward and can be found either in the DMM manufacturer specifications or calibration certificate. However, when using a current transducer along with a measurement instrument, or when using a current output current transducer with a burden resistor and a voltage measuring instrument, more analysis is needed.

This application note is a tutorial on how to perform uncertainty analysis in the context of current measurements using current transducers and some measurement instruments. This is not a thorough treatment of uncertainty analysis, as this is a very broad topic, but rather an introduction to get the reader thinking about their measurement and setup. The reader is referred to the following books for a more comprehensive treatment of uncertainties in measurements [1,2], and the ISO Guide to the Expression of Uncertainty Measurement (GUM) [3].

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Equipment

- Current Transducer with Current or Voltage output
- Measurement Instrument such as a Digital Multimeter (DMM) or a Power Analyzer

Applications

- Current Measurement
- Calibrating Shunts
- Calibrating Battery Cyclers
- Measuring Electrical Power Conversion Efficiency

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Introduction

Measurement is the process of determining the *best estimate* $\bar{X}$ of a quantity of interest X, referred to as the *measurand*. By best estimate we imply that there is some *uncertainty* $u(X)$ in the measurement. The instruments are not perfect and there is noise from different parts of the measurement setup; these issues reduce our ability to measure precisely. The true value of the measurand lies somewhere in the interval $\bar{X} \pm u(X)$. Note that the *measurand and its uncertainty have the same units*. We need to quantify how good our best estimate is, i.e. we need to estimate the uncertainty of the measurement. There are often multiple ways or instruments that can be used to perform the same measurement; however, they will often result in different uncertainty and ultimately, we need to balance the practicality and cost of a measurement setup with its uncertainty.

We classify the sources of uncertainty of a measurement into Type A and Type B. *Type A* uncertainty is due to the statistical nature of the measurement. If we are measuring the current I with a DMM and take 10 different measurements, we will obtain 10 different values; the standard deviation of these values is Type A uncertainty $u_A(I)$. *Type B* uncertainty refers to uncertainty determined by other means; the DMM we used to measure the current has an accuracy specification from which we can infer Type B uncertainty $u_B(I)$. In this example both types of uncertainty contribute to the measurement uncertainty that we refer to as *combined uncertainty* and is the quadrature sum of the sources of uncertainties $u_C(I) = \sqrt{u_A^2(I) + u_B^2(I)}$. This uncertainty number represents a standard deviation (assuming the distribution is gaussian), and typically we take two standard deviations as the uncertainty estimate; this number is referred to as *expanded uncertainty* $u(I) = k\, u_B(I)$, where $k = 2$ corresponding to 95% coverage. An instrument accuracy specification or calibration certificate uncertainty number is effectively an expanded uncertainty and is converted into Type B uncertainty dividing by $k = 2$.

Some measurements are direct in that we use one instrument that directly yields data points of the measurand. Some examples include measuring the length of an object with a ruler or the current on a conductor with a DMM in series; the ruler directly yields the length and the DMM the current. Other measurements are indirect in that we need several instruments to obtain data points of different sub-quantities, or convert a quantity or its scale, and the results are plugged into a mathematical expression to obtain data points for the measurand. Measuring current with a current transducer and a DMM is one such example; the current transducer converts the primary current of interest into a different quantity (current or voltage at a different scale) that is then measured by the DMM.

Estimating uncertainty in a direct measurement is a relatively straightforward process as previously discussed in the example of measuring current directly with a DMM. Estimating uncertainty in an indirect measurement is a more mathematically involved process. We begin by formulating a mathematical model of the measurement $Y = f(X_1, \dots, X_m)$, where Y is the measurand and $X_1, \dots, X_m$ are the different sub-quantities we measure directly. Each sub-quantity has its own combined uncertainty, and we need to estimate the contribution of each to the measurand. We do this by linearizing the measurement model around the best estimate point $\bar{Y} = f(\bar{X}_1, \dots, \bar{X}_m)$. We analytically calculate the sensitivity coefficient of each sub-quantity that is the partial derivative of the measurand with respect to the sub-quantity $\partial Y / \partial X_i$, and evaluate them at the point of best estimate; the combined uncertainty of the measurand is then the quadrature sum of the combined uncertainty of each sub-quantity weighted by the respective sensitivity coefficient.

In the following we show how to perform uncertainty analysis in the case of measuring current with a current transducer and a measurement instrument such as a DMM or Power Analyzer; the methodology is applicable to both current and voltage output transducer, and both DC and AC measurements. We begin by deriving a mathematical model of the measurement. We then analyze the sources of uncertainty starting from the sub-quantities, we calculate the sensitivity coefficients, and assemble the final expressions for the combined and expanded uncertainty of the measurement. Finally, a numerical example with simulated data is presented. The appendices provide a starting point for the reader in refining the measurement model for certain cases.

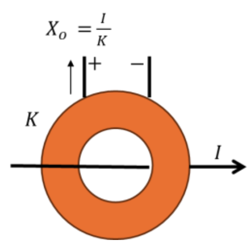

Figure 1: Current Measurement with a Current Transducer Description.

Measurement Definition

Figure 1 summarizes the measurement setup. A conductor carrying current I goes through the primary of the current transducer with a current or voltage output, having a rated ratio K_r, actual ratio K, and transformer ratio error:

$$\epsilon = \frac{K_r - K}{K}. \quad (1)$$

We need to measure the current I going through the primary of the transducer by measuring the transducer's output current or voltage X_o using an appropriate instrument, such as a DMM or Power Analyzer. The primary current I can be calculated from the measured transducer output X_o as follows:

$$I = KX_o. \quad (2)$$

The calibration certificates and manufacturer specification sheets will typically report the transformer ratio error ϵ instead of the true ratio K, and therefore we need to express (2) as a function of ϵ. Solving (1) for K yields:

$$K = \frac{K_r}{1+\epsilon}, \quad (3)$$

Plugging (3) into (2) yields the *model of the measurement*:

$$I = \frac{K_r X_o}{1+\epsilon}. \quad (4)$$

The measurement of the primary current is therefore performed indirectly by taking a series of n measurements of the output of the current transducer $X_o^{(i)}$. Then the best estimate for the output of the transducer is:

$$\overline{X_o} = \langle X_o^{(i)} \rangle.$$

(5)

Plugging (5) into (4) yields the best estimate of the primary current as a function of the transducer's rated ratio, ratio error, and measured output:

$$\bar{I} = \frac{K_r}{1+\epsilon}\overline{X_o}.$$

(6)

If the ratio error ϵ is sufficiently small, we can often approximate the best estimate of the primary current as:

$$\bar{I} \approx K_r\overline{X_o}.$$

(7)

Uncertainty Analysis

Now that we have established the measurand with equation (6), we can estimate the uncertainty of the measurement. The sources of uncertainty are (a) combined uncertainty of the measurement of the transducer's output $u_C(X_o)$, and (b) Type B uncertainty of the error ratio of the transducer $u_B(\epsilon)$. We assume that all sources of uncertainty are uncorrelated.

The sources of uncertainty in the measurement of the transducer output X_o are Type A due to the statistical nature of the measurement:

$$u_A^2(X_o) = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_o^{(i)} - \overline{X_o}\right)^2,$$

(8)

and Type B due to the uncertainty of the meter used to measure $X_o^{(i)}$, denoted by $u_B(X_o)$, assuming that the quantity X_o is measured directly on the meter (i.e. X_o does not represent a current measured indirectly as a voltage through a burden resistor). The combined uncertainty for the transducer output is:

$$u_C(X_o) = \sqrt{u_A^2(X_o) + u_B^2(X_o)}.$$

(9)

The sources of current measurement uncertainty need to be propagated through the sensitivity coefficients described below from equation (4):

$$\frac{\partial I}{\partial X_o} = \frac{K_r}{1+\epsilon},$$

(10)

$$\frac{\partial I}{\partial \epsilon} = -\frac{K_r X_o}{(1+\epsilon)^2}.$$

(11)

Software packages such as Mathematica [4] or Wolfram Alpha [5] can be used to calculate analytically these partial derivatives. The combined uncertainty of the primary current is:

$$u_C(I) = \sqrt{\left(\frac{\partial I}{\partial X_o}\right)^2 u_C^2(X_o) + \left(\frac{\partial I}{\partial \epsilon}\right)^2 u_B^2(\epsilon)}\,,$$

(12)

And the expanded uncertainty is:

$$u(I) = k\, u_C(I),$$

(13)

where typically $k = 2$ for 95% coverage.

Instrument Uncertainties & Setup

There are two options to obtain the instrument uncertainties, i.e. $u_B(X_o)$ and $u_B(\epsilon)$: a) the manufacturer specifications, or b) a calibration certificate.

Manufacturer specifications typically have the format of $\%\,Reading + \%\,Range$, where we are given two numbers, a fractional multiplier of the reading $\epsilon_{reading}$ and a fractional multiplier of the range ϵ_{range}. Assuming we are measuring a quantity Y with a certain instrument; we are operating the instrument in the Y_{range} range and the reading is $Y_{reading}$. The associated uncertainty is:

$$u(Y) = \epsilon_{reading} Y_{reading} + \epsilon_{range} Y_{range},$$

(14)

where the fractional multipliers, if given as percentages have been converted to absolute fractions by dividing by 100, and if given as parts-per-million (ppm) by dividing by 10^6. The resulting number represents an expanded uncertainty and therefore needs to be converted into a Type B uncertainty to be used in evaluating (12) by dividing by $k = 2$, i.e. $u_B(Y) = u(Y)/k$.

Calibration Certificates typically directly report the expanded uncertainty of a given instrument range and therefore needs to be converted into a Type B uncertainty to be used in evaluating (12) by dividing by $k = 2$, i.e. $u_B(Y) = u(Y)/k$.

Note: when an instrument has multiple ranges, i.e. a DMM or a Power Analyzer, it is imperative that we use the uncertainties corresponding to the active range of the instrument. To ensure this is the case, it is often better to manually set the range on the instrument than rely on the *Autorange* feature. It is also important to properly set up the instrument to improve the accuracy of the measurement. For example, when using a DMM, to take full advantage of the digits of the DMM a large Number of Power Line Cycles (NPLC) needs to be set, or when performing AC measurements with a Power Analyzer, the acquisition time needs to be much larger than the fundamental period of the signal.

Numerical Example

We are trying to measure approximately $3000\,A$ of DC current using a current output current transducer with a rated ratio $K_R = 1500$ and a 6.5-digit DMM. From the transducer's calibration certificate $\epsilon = -23\,ppm = -2.3 \cdot 10^{-5}$ and $u(\epsilon) = 0.1\% = 10^{-3}$, which corresponds to $u_B(\epsilon) = u(\epsilon)/2 = 0.5 \cdot 10^{-3}$. We set the DMM in the $3\,A$ range. From the DMM's calibration certificate for the $3\,A$ range $u(X_o) = 3 \cdot 10^{-4}\,A$, which corresponds to $u_B(X_o) = u(X_o)/2 = 1.5 \cdot 10^{-4}\,A$.

We take 10 measurements with the DMM that have a mean value $\overline{X_o} = 1.9995\,A$ and sample standard deviation $u_A(X_o) = 2.9 \cdot 10^{-4}\,A$. Plugging the numbers into (4) we get the best estimate of the primary current $\bar{I} = 2999.32\,A$.

Plugging numbers into (9) we get the combined uncertainty for the transducer output $u_C(X_o) = 3.3 \cdot 10^{-4}\,A$. Plugging numbers into (10) and (11) we get the sensitivity coefficients $\frac{\partial I}{\partial X_o} = 1500.03$ and $\frac{\partial I}{\partial \epsilon} = 2999.39\,A$. Plugging numbers into (12) yields the combined uncertainty of the primary current $u_C(I) = 1.58\,A$, which corresponds to an expanded uncertainty $u(I) = 3.15\,A$. Therefore, the true primary lies in the interval $2999.3 \pm 3.1\,A$.

Appendix 1 - DC Offset Correction

When measuring DC current, current transducers may exhibit a small zero current offset. We can adapt the measurement model of (4) to capture this effect as follows:

$$I = \frac{K_r(X_o - X_{off})}{1+\epsilon},$$

(15)

where X_{off} is the output of the current transducer when the primary current is zero, i.e. when the circuit is open. We have added an additional source of uncertainty: the combined uncertainty of the measurement of the transducer's output at zero primary current $u_C(X_{off})$.

We need to repeat the Uncertainty Analysis with the new model of the measurement and re-calculate the sensitivity coefficient of each source of uncertainty and the combined uncertainty of the primary current; this is left as an exercise to the reader.

Note that the zero current offset can be different after each power cycle of the transducer or after a measurement overload. For measurement of current amplitudes that are small compared to the transducer range, it may be important to measure the zero offset at the beginning and at the end of the measurement.

Appendix 2 – Measuring Current with a Burden Resistor

Often the voltage channel of a DMM has better accuracy than the current channel and it may be preferrable to use a current output current transducer with a burden resistor and measure the primary current indirectly by measuring the voltage V_o across the burden resistor R_B. Substituting $X_o = V_o/R_B$ in (4) yields the new measurement model:

$$I = \frac{K_r}{1+\epsilon}\frac{V_o}{R_B}.$$

(16)

We have traded the uncertainty of measuring current with the uncertainty of measuring voltage and have added an additional source of uncertainty: the Type B uncertainty of the burden resistor value $u_B(R_B)$.

We need to repeat the Uncertainty Analysis with the new model of the measurement and re-calculate the sensitivity coefficient of each source of uncertainty and the combined uncertainty of the primary current; this is left as an exercise to the reader. We can further improve the model of (16) for DC measurements by adding the zero current offset term. The derivation is left as an exercise to the reader.

References

[1] Taylor, J. R. (1997). An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements.

[2] Crowder, Stephen, et al. Introduction to statistics in metrology. Cham, Switzerland: Springer, 2020.

[3] https://www.bipm.org/en/publications/guides/

[4] https://www.wolfram.com/mathematica/

[5] https://www.wolframalpha.com/