Application Note: Transducer Termination Impedance – Low and High Frequency Analysis

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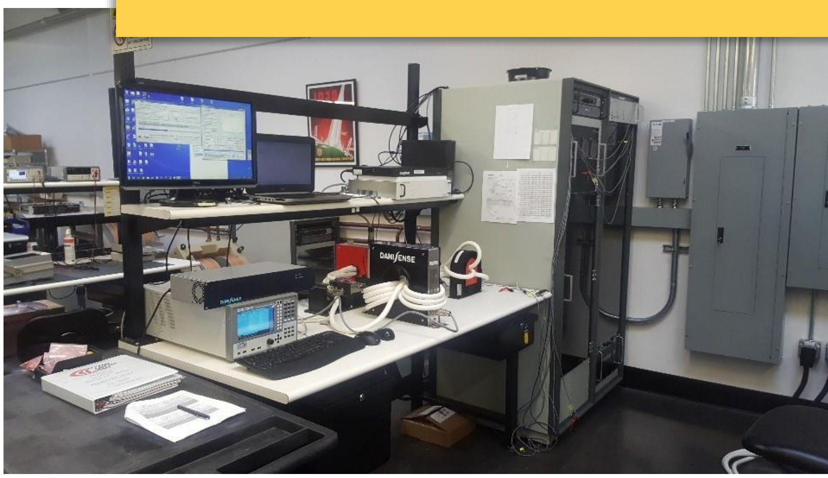

Equipment

- Current or Other Transducer with Voltage Output
- Measurement Instrument such as an Oscilloscope

Overview

Voltage output transducers need to be properly terminated on the measurement instrument according to the manufacturer's specifications for accurate measurements, and for some products, to avoid damaging the transducer. Different products, even from the same manufacturer, may have different termination specifications and it is therefore critical to *check the specifications and not assume*.

We often receive questions about a factor of two discrepancies in measurements because of improper termination, and about reflections on the coaxial cables when terminating into high impedance. There is often confusion between the source impedance and termination impedance. This application note analyses the problem of signal propagation inside a coaxial cable under different termination conditions to gain insight into these issues. The analysis is carried out both using lumped elements (low-frequency) and transmission lines (high-frequency). Transmission line theory is briefly introduced; we assume the reader understands circuit theory.

Applications

- Current Measurement with Voltage Output Transducers.

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Introduction

Figure 1 shows the schematic of the problem we need to analyze. A current transducer with a voltage output BNC connector and $50\,\Omega$ source impedance can be modeled as an ideal voltage source with zero impedance and a series $50\,\Omega$ resistor. The output of the transducer is connected to a voltage measurement instrument such as a Digital Multimeter (DMM) or an Oscilloscope. A DMM typically has a high impedance, often greater that $1\,\mathrm{M}\Omega$. Oscilloscopes typically have an internal $1\,\mathrm{M}\Omega$ termination impedance, with higher end oscilloscopes also having the option for $50\,\Omega$. We need to analyze the behavior of this circuit under different termination impedance of the measurement instrument, $1\,\mathrm{M}\Omega$ or $50\,\Omega$.

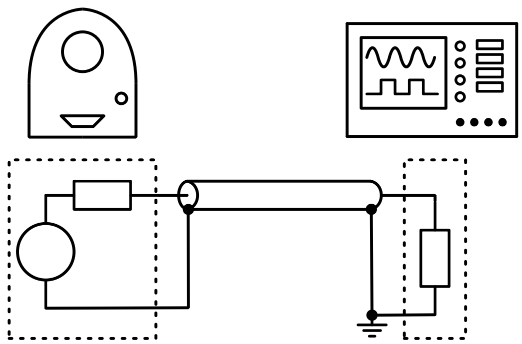

Figure 1: Problem overview schematic.

A question that often comes up is whether this should be viewed as a transmission line problem, and therefore what happens with reflections; the short answer for current transducers in a typical lab environment is that it doesn't matter, as long as the manufacturer's termination specifications are observed. As will be shown in subsequent sections, lumped element (circuit) analysis and transmission line theory result in the same behavior as far as the measurement is concerned.

Table 1: Free-space wavelength corresponding to different frequencies.

Frequency	Free-Space Wavelength	Notes
60 Hz	5,000 km	AC power distribution freq in North America.
1 kHz	300 km	
75 kHz	4 km	Typical power converter switching frequency.
1 MHz	300 m	
10 MHz	30 m	Highest frequency **Danisense** transducer.
50 MHz	6 m	Highest frequency **PEM** Rogowski Coil.
500 MHz	60 cm	Highest frequency **MagneLab** CT.
1.5 GHz	20 cm	Highest frequency **Bergoz** FCT.
2.45 GHz	12 cm	Typical microwave oven magnetron frequency.

In setting the stage for the analysis we need to understand the electrical size or length of the problem, i.e. how the physical size of the system relates to the wavelength or the frequency of operation. Table 1 shows the free-space wavelength corresponding to several frequencies. If multiple frequencies are involved, i.e. as in a switching converter, then we either need to look at the highest frequency of interest (or carrying substantial power) or look at the problem at each frequency. To first order, we can identify three regions: a) the wavelength is very large compared to the physical size of the problem where we can apply Lumped Element Analysis, b) the wavelength is comparable to the physical size of the problem where we need to use Transmission Line Theory, and c) the wavelength is much smaller than the physical size of the problem where we have to use either Transmission Line Theory or resort to numerical methods. These three regions, along with the typical method of analysis, are graphically illustrated in Figure 2. Note that this classification is an oversimplification: the analysis is determined by the features of the specific problem at hand, and in a complex system it may involve all three methods. Transmission lines can be modelled and/or constructed with lumped elements, extremely large problems can often be decomposed into sections of transmission lines, and typically numerical methods are used when the geometric features of the problem are small compared to the wavelength, i.e. the matching irises in a waveguide magic-Tee.

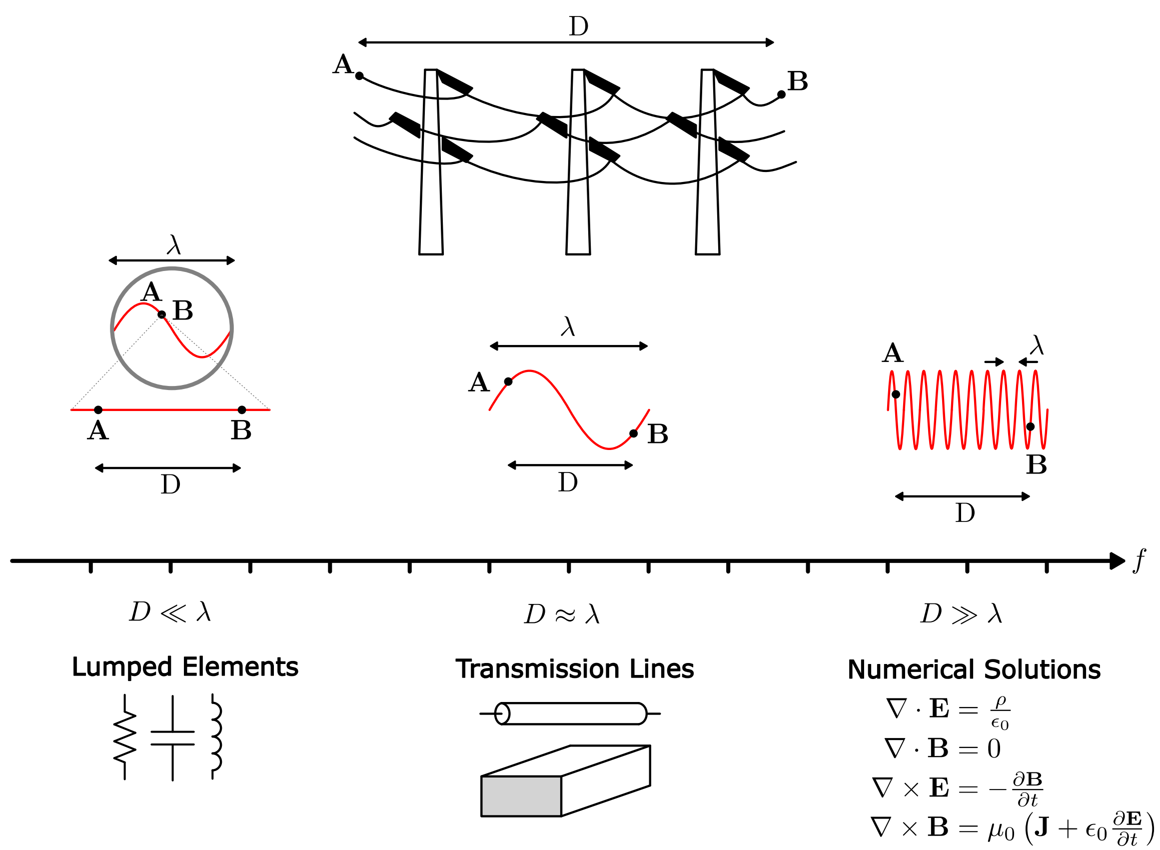

Figure 2: Broad classification of analysis methodology of the different electrical sizes.

Some definitions: **Lumped Element or Circuit Analysis** refers to a mathematical approximation to Maxwell's equations where the problem can be decomposed into ideal electrical elements such as resistors, capacitors, and inductors, the qualities we look at are voltage and current, and the finite speed of light is ignored, i.e. electrical signals propagate instantly from the node of one circuit element to the other. Although originally derived separately, Kirchoff's Voltage and Current Laws can be derived from Maxwell's equations. Lumped element analysis is valid when the circuit dimensions are very small compared to the wavelength. We assume that the reader understands lumped element (circuit) analysis.

Transmission Line Theory refers to another mathematical approximation to Maxwell's equations where the problem can be decomposed into transmission lines of different impedances connected together and with lumped elements. Electromagnetic waves propagate in these transmission lines with finite speed and any junction can cause reflections. Transmission Line Theory analysis is typically performed when the size of the problem is comparable or large compared to the wavelength, although as will be shown later, Transmission Line Theory will yield the same results as Circuit Analysis when applied to electrically small problems. When the problem is geometrically complex, i.e. small geometrical features compared to the wavelength, we need to resort to numerical solutions of Maxwell's equation; this is not relevant to this Application Note.

This Application Note is organized as follows: We begin by briefly reviewing Transmission Line Theory. We proceed to analyze the $50\,\Omega$ and high-impedance termination using both Transmission Line and Circuit analysis. We then provide some additional considerations and conclude with an overview of how different current transducers GMW carries should be terminated.

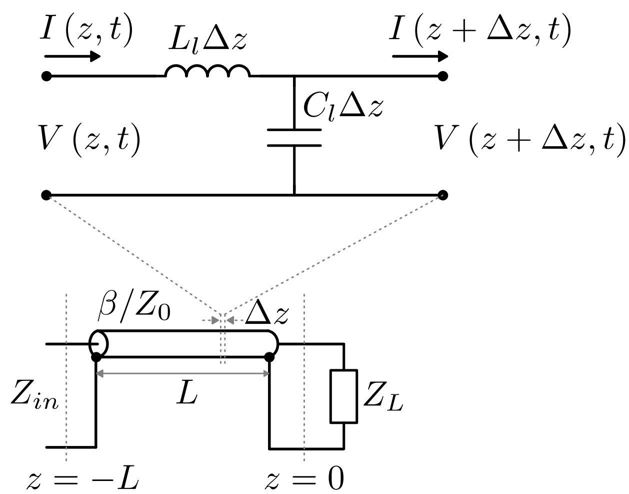

Figure 3: Terminated transmission line schematic and circuit model of an infinitesimal segment.

Transmission Line Theory

Generally, the term **Transmission Line** refers to a physical structure that guides Electromagnetic waves without reflection or mode conversion[1], except for terminations and change of impedance, and maintains the same wave propagation velocity. Examples of physical transmission lines include coaxial cables and waveguides, twisted cable pairs, metallic rectangular or cylindrical waveguides, and optical fibers. Transmission Line theory is a mathematical approximation to Maxwell's equations where a Transmission Line model becomes a circuit element, like resistors/capacitors/inductors in circuit theory. The properties of a Transmission Line model are its wave impedance Z_0, referred to simply as impedance,

[1] Transmission lines such as coaxial cables and waveguides are specified to operate at a frequency range where only the fundamental field configuration, or mode, propagates; however, there are higher order modes that can propagate at higher frequencies. If operated at higher frequencies that specified, any bends or imperfections will cause power to convert from one mode to another and the line will not be useful.

the propagation constant β, and its length L. In this note we ignore the attenuation and dispersion (dependence of β on the frequency).

Deriving the model of a transmission line involves deriving the wave equation of the line; this can either be initiated from Maxwell's equations where we derive the electric and magnetic fields wave equations for a specific physical structure, or alternatively discretizing the transmission line into a lumped element (circuit) model where we derive the voltage and current wave equations given the inductance per unit length L_l and capacitance per unit length C_l of the line. The former method is typically used in waveguides or optical fibers, while the latter is typically used in cables, such as coaxial or twisted pair cables, but the results and subsequent steps will be the same in either case.

In this Application Note we will skip the derivation of the wave equation; the reader is referred to the following classic Microwave Engineering books for a more in-depth analysis and derivations [1,2]. For the rest of this Note we will adopt the circuit model approach of transmission lines. Figure 3 shows a terminated transmission line of impedance Z_0, propagation constant β, and length L. We discretize this transmission line into infinitesimal segments of length $\Delta z \to 0$; each such segment is model as a lumped LC low pass filter.

If we derive and solve the wave equations of the voltage and current, we get solutions of the form:

$$V(z,t) = V_0^+ e^{j(2\pi ft - \beta z)} + V_0^- e^{j(2\pi ft + \beta z)}$$
$$I(z,t) = I_0^+ e^{j(2\pi ft - \beta z)} - I_0^- e^{j(2\pi ft + \beta z)} \text{ ,}$$

(1)

where the terms $e^{j(2\pi f - \beta z)}$ correspond to forward waves, the terms $e^{j(2\pi f + \beta z)}$ correspond to backward waves, the propagation constant is

$$\beta = \frac{2\pi}{\lambda_g},$$

(2)

λ_g is the guided wavelength inside the transmission line, and V_0^+, V_0^-, I_0^+, I_0^- are the amplitudes of the respective waves. The phase velocity in the transmission line is

$$v = f\lambda_g = 2\pi f \frac{\lambda_g}{2\pi} = \frac{\omega}{\beta} = \frac{1}{\sqrt{L_l C_l}}.$$

(3)

Coaxial cables at relatively low frequencies below 100 MHz have no dispersion and the phase velocity is effectively equal to the group velocity, i.e. the velocity with which the overall envelope shape of the signal propagates. At higher frequencies, cables may exhibit dispersion; we ignore dispersion in this Note.

The voltage and current wave amplitudes are related through the impedance of the transmission line

$$Z_0 = \frac{V_0^\pm}{I_0^\pm} = \sqrt{\frac{L_l}{C_l}}.$$

(4)

Note the minus sign in front of the term I_0^- in (1); this is to mathematically set the power flow direction. To understand this, we need to look at the electric and magnetic fields and the Poynting vector

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H},$$

(5)

where $\boldsymbol{S}$ is a vector giving the direction and rate of energy transfer at a point in space, $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field vectors at that point, and $\times$ is a vector cross product; we will look at an example of a coaxial line illustrated in Figure 4. The voltage, and therefore electric field, has the same polarity in both forward and backwards-propagating waves; however, the current, and therefore magnetic field has opposite polarity according to (1). Calculating the Poynting vector from (5) with the right-hand rule yields the power flow direction.

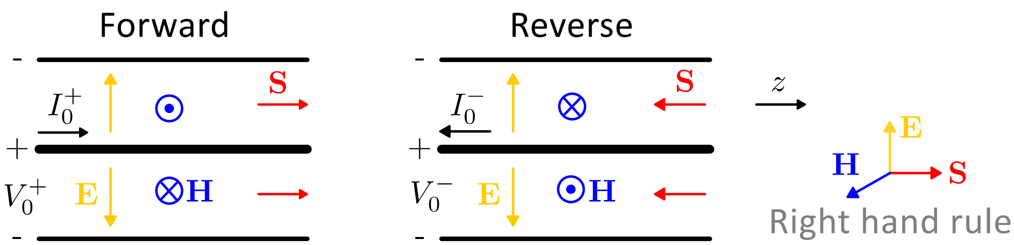

Figure 4: Power flow in a coaxial line.

We will now drop the time variation and treat the voltage and current as *phasors*. At the load $z = 0$, the voltage and current phasors are

$$V_L = V(0) = V_0^+ + V_0^-$$
$$I_L = I(0) = I_0^+ - I_0^- = \frac{1}{Z_0}(V_0^+ - V_0^-)'$$

(6)

where we used equation (4). We can relate V_L and I_L through the load impedance

$$Z_L = \frac{V_L}{I_L} = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}.$$

(7)

We can rearrange terms in (7) to get the reflection coefficient Γ_L at the load:

$$Z_L(V_0^+ - V_0^-) = Z_0(V_0^+ + V_0^-) \rightarrow$$

$$V_0^+(Z_L - Z_0) = V_0^-(Z_L + Z_0) \rightarrow$$

$$\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

(8)

Plugging equation (8) into (1) yields the voltage phasor across the line

$$V(z) = V_0^+ e^{-j\beta z} + \Gamma_L V_0^+ e^{j\beta z} \rightarrow$$

$$V(z) = V_0^+ e^{-j\beta z}\left(1 + \Gamma_L e^{j2\beta z}\right).$$

(9)

It follows from (9) that when the reflection coefficient is not zero, a standing wave is formed and the resulting *voltage across the line is a function of the position*.

We can now calculate the impedance seen from the input side of the transmission line:

$$Z_{in} = \frac{V(-L)}{I(-L)} = Z_0 \frac{V_0^+ e^{j\beta z} + \Gamma_L V_0^+ e^{-j\beta z}}{V_0^+ e^{j\beta z} - \Gamma_L V_0^+ e^{-j\beta z}} = Z_0 \frac{e^{j\beta z} + \Gamma_L e^{-j\beta z}}{e^{j\beta z} - \Gamma_L e^{-j\beta z}} \rightarrow$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)}.$$

(10)

It follows from (10) that the impedance seen from the input of the transmission line changes both with the termination and line impedance, and length of the transmission line. It also follows from (10) that by cascading transmission lines of different lengths and impedances we can convert the impedance of a load to any impedance value; this is how impedance matching is often performed at very high frequencies [1,2]. Another consequence of (10) is that a very short discontinuity in the impedance, modelled as a short transmission line of some impedance with $\beta L \rightarrow 0$ connected to a load impedance Z_L, will be transparent:

$$\lim_{\beta L \rightarrow 0} Z_{in} = Z_L,$$

(11)

for example, when using a BNC-to-Banana connector that is short electrically compared to the operating wavelength $\beta L \rightarrow 0$, or when using a long cable but the operating wavelength is very short, e.g. 60 Hz or 5,000 km wavelength and 100 m cable.

Now that we have laid out the basic physics of transmission lines, we can apply them to analyze the problem at hand – terminating a transducer into a measuring instrument.

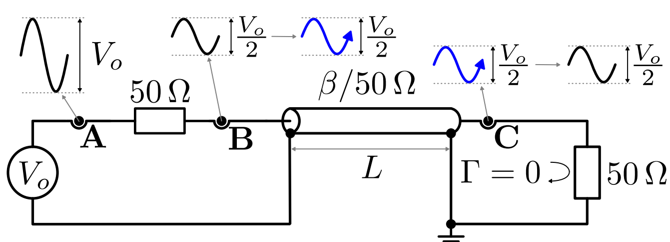

Figure 5: Transmission line model of a voltage output transducer terminated into $50\,\Omega$ through a $50\,\Omega$ transmission line.

Terminating into 50 Ohm

When terminating a voltage-output transducer with $50\,\Omega$ source impedance into a $50\,\Omega$ load through a $50\,\Omega$ transmission line, then the measured voltage is half, translating to the **sensitivity of the transducer being halved**. Both transmission line and lumped element modeling yield the same result.

Transmission Line Analysis

With reference to Figure 5, an ideal voltage source with $50\,\Omega$ source impedance generates a voltage V_0. The $50\,\Omega$ transmission line terminated into a $50\,\Omega$ load appears as a $50\,\Omega$ load at point $\mathbf{B}$. The $50\,\Omega$ source impedance forms a resistive divider with the $50\,\Omega$ equivalent transmission line impedance seen from point $\mathbf{B}$, and as a result the voltage at point $\mathbf{B}$ is $V_0/2$. This voltage launches a wave of amplitude $V_0^+ = V_0/2$ inside the transmission propagating towards the load. The wave reaches the $50\,\Omega$ load and gets absorbed without any reflections, since the line is matched to the load. The measured voltage at point $\mathbf{B}$ is $V_0/2$. Since the output of the transducer is proportional to its sensitivity and the output is halved, the sensitivity of the transducer appears to be half.

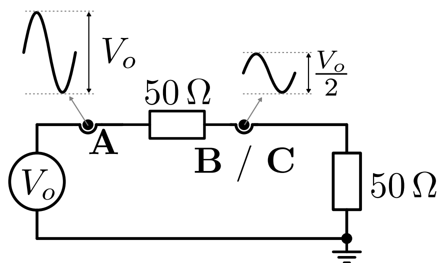

Figure 6: Lumped elements model of a voltage output transducer terminated into $50\,\Omega$.

Lumped Element Analysis

With reference to Figure 6, the transmission line of Figure 5 is removed and replaced with a short circuit. The $50\,\Omega$ source impedance forms a resistive divider with the $50\,\Omega$ termination impedance and as a result the measured voltage at point $\mathbf{B}$ is $V_0/2$, and the sensitivity of the transducer appears to be half.

Output Drive Issues

Active transducers have an output drive amplifier; driving a $50\,\Omega$ load requires more drive current than a high impedance load. In some products the output amplifier cannot provide enough current and may either affect the accuracy of the measurement or damage the transducer. In some other products the output amplifier can drive a $50\,\Omega$ but not for the full measurement range of the transducer. **Always check the datasheet and manual of the specific transducer model to ensure operation according to the manufacturer's specification for best accuracy and to avoid damaging the transducer.**

Terminating into High-Impedance

When terminating a voltage-output transducer with $50\,\Omega$ source impedance into a high impedance, e.g. $1\,\mathrm{M}\Omega$, load through a $50\,\Omega$ transmission line, then the full voltage is measured, and the transducer sensitivity is nominal. Both transmission line and lumped element modeling yield the same result. Transmission line analysis further shows that a standing wave is formed in the transmission line that does not affect the measurement.

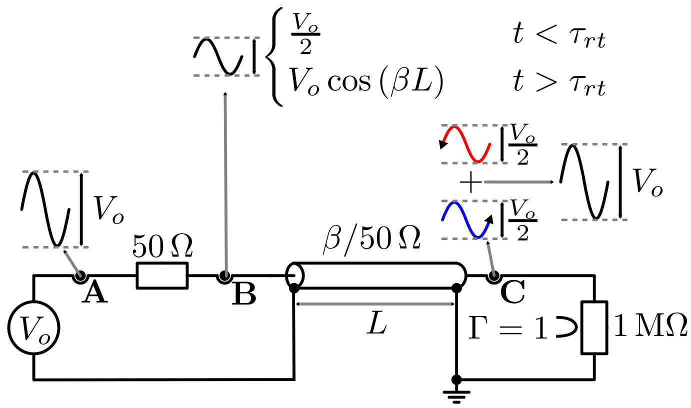

Figure 7: Transmission line model of a voltage output transducer terminated into $1\,M\Omega$ through a $50\,\Omega$ transmission line.

Transmission Line Analysis

With reference to Figure 7, an ideal voltage source with $50\,\Omega$ source impedance generates a voltage V_0. Initially, during time $t < \tau_{rt}$, where

$$\tau_{rt} = \frac{2L}{v}$$

(12)

is the round-trip time of the transmission line, the $50\,\Omega$ source impedance forms a resistive divider with the $50\,\Omega$ transmission line and as a result the voltage at point $\mathbf{B}$ is $V_0/2$. This voltage launches a wave of amplitude $V_0^+ = V_0/2$ inside the transmission propagating towards the load. After half round-trip time the wave reaches the high impedance and from (8) $\Gamma_L = 1$ and the wave gets fully reflected, i.e. $V_0^- = V_0/2$. The measured voltage at point $\mathbf{C}$ is the sum of both forward and reflected waves $V_0^+ + V_0^- = V_0$. As a result, the measured voltage at point $\mathbf{C}$ is practically V_0 and the sensitivity of the transducer appears nominal. The reflected wave V_0^- reaches point $\mathbf{B}$ after time $t > \tau_{rt}$ forming a standing wave in the transmission line; the reflected wave gets fully absorbed by the $50\,\Omega$ source impedance (the ideal voltage source has zero impedance) and from (9) the voltage at point $\mathbf{B}$ becomes

$$V(-L) = V_0^+ e^{j\beta L}\left(1 + e^{-j2\beta L}\right) = \frac{V_0}{2} 2\cos(\beta L) = V_0 \cos(\beta L).$$

(13)

Despite this standing wave that perturbs the voltage throughout the transmission, the measured voltage is unaffected.

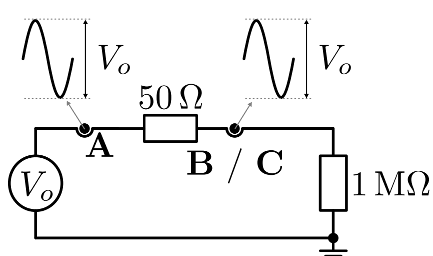

Figure 8: Lumped elements model of a voltage output transducer terminated into $1\,M\Omega$.

Lumped Element Analysis

With reference to Figure 8, the transmission line of Figure 7 is removed and replace with a short circuit. The $50\,\Omega$ source impedance forms a resistive divider with the $1\,\text{M}\Omega$ termination impedance and as a result the measured voltage at point $\mathbf{B}$ is practically V_0 while the sensitivity of the transducer appears nominal.

Miscellaneous Considerations

Oscilloscope termination impedance: Mid and low-end oscilloscopes typically have high impedance inputs. High-end oscilloscopes often have both a $50\,\Omega$ and high-impedance input option that can be changed by the user, while very high bandwidth oscilloscopes (> 2 GHz) only have a $50\,\Omega$ inputs. If using an oscilloscope with only high-impedance termination and need to terminate a transducer into $50\,\Omega$, $50\,\Omega$ *BNC feedthrough terminations* can be purchased and installed between the cable output and oscilloscope input. Alternatively, a BNC Tee can be used along with a $50\,\Omega$ terminator on one port. If using a very high bandwidth oscilloscope with only $50\,\Omega$ termination and need to terminate a transducer into high-impedance, the only option is to purchase a high-impedance adapter from the oscilloscope manufacturer that is an active device.

Bandwidth & time-domain specifications guarantee: Some transducers can be terminated both into high-impedance and $50\,\Omega$ with half the sensitivity; however, the manufacturer will guarantee the bandwidth and time-domain specifications in only one of the two configurations. For example, MagneLab only guarantees specifications when terminating into $50\,\Omega$, while Danisense only guarantees specifications when terminating into $10\,\text{M}\Omega$. In these cases, it is important to select the termination that guarantees the specifications that are crucial for the specific measurement.

Non $50\,\Omega$ cables and connectors: BNC connectors and cable assemblies come in both $50\,\Omega$ (e.g. RG 58 cable) and $75\,\Omega$ (e.g. RG 59 cable). If the shortest wavelength of interest is much larger than the length of the cable, then the measurements will not be affected; however, if that is not the case the impedance mismatch will be detrimental to the accuracy of the measurement.

Connector quality and aging: If operating at very high frequencies above 100 MHz, the quality of the cables and connectors may start affecting the measurement, especially as metallic debris on the connector dielectric from connection/disconnection cycles may start inducing reflections. It is good practice to visually inspect the connectors and clean them, especially if often connected/disconnecting. Higher quality cables, such as Times Microwave LMR240 or LMR400 (for long cables), may be needed.

Key Points and Current Transducer Summary

- **Always check the datasheet and manual of the specific transducer model to ensure operation according to the manufacturer's specification for best accuracy and to avoid damaging the transducer – do not assume!**
- **Provided that is allowed by the manufacturer, terminating into $50\,\Omega$ results in half the sensitivity compared to terminating into high impedance.**
- When the shortest wavelength of interest is much larger than the length of the cable, the coaxial cable is effectively a short circuit and there is no reflection concern.

- When the shortest wavelength is comparable or shorter than the length of the cable, then transmission line effects are present; if terminating into high impedance then a standing wave will be present but that does not affect the measurement.
- Danisense Voltage-output DCCTs: Typically, 1V output transducers can be terminated into both high impedance and $50\,\Omega$, while 10V output transducers can only be terminated into high impedance. Check that the source impedance of the specific model in the datasheet is $50\,\Omega$ before terminating into $50\,\Omega$. Danisense only guarantees the bandwidth and time domain specifications if terminating into $10\,\mathrm{M}\Omega$.
- PEM Rogowski Coils: Generally, PEM Rogowski Coils cannot be terminated into $50\,\Omega$ and must be terminated in high impedance. Especially for the LFR series, there is risk of damage if terminating into $50\,\Omega$. At the time of this writing, the Ultra Mini coils can also be terminated into $50\,\Omega$ with half the sensitivity; however, the measurement range is reduced.
- MagneLab CTs: MagneLab CTs can be terminated into both high impedance and $50\,\Omega$. MagneLab only guarantees the bandwidth and time domain specifications if terminating into $50\,\Omega$.
- GMW CPC/CPCO: GMW voltage output probes must be terminated into high impedance.
- Bergoz FCTs: FCTs must be terminated into $50\,\Omega$.

References

[1] Pozar, David M. Microwave engineering: theory and techniques. John Wiley & Sons, 2021.

[2] Collin, Robert E. Foundations for microwave engineering. John Wiley & Sons, 2007.