The Physics of Current Measurement

Presented by:

Filippos Toufexis, PhD
Applications Engineer
GMW Associates
filippos@gmw.com

GMW Associates

Agenda

1 **GMW Overview**

- Current Transducers & Calibration Services
- Magnetic Field Transducers & Calibration Services
- Electromagnet Systems

2 **Current Transducer Technologies**

- Passive Current Transformers
- Active Current Transformers
- Rogowski Coils
- Flux-gate DC Current Transformers
- Hall Effect Current Sensors

3 **Common Issues**

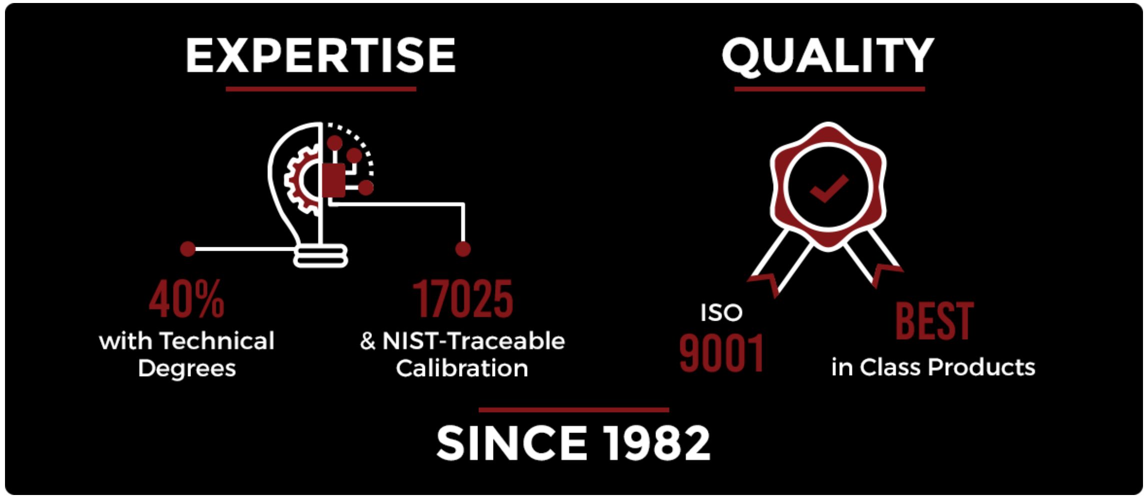
- Transducer Termination Impedance
- Uncertainty Analysis

GMW Associates

GMW Associates – Overview

GMW is the designer, integrator, and distributor of Magnetic Systems and Instrumentation based on Magnetics

- Founded in 1982
- Staff of 20, 50% with technical degrees
- Headquarters in San Carlos, California (30 miles from San Francisco)
- Background in Accelerator Physics, MRI, Instrumentation, Materials Research, and Power Electronics



Instrumentation

- Electric Current Measurement
- Magnetic Field Measurement
- Particle Beam Diagnostics

Calibration and Service

- 17025 Accredited Current Transducer Calibrations
- Magnetic Field Mapping
- Magnetic Site Survey

Electromagnet Systems

- Dipole Magnets
- Projected Field Magnets
- High-Uniformity Magnets
- Magnetic Modeling & Design

GMW Current Transducers & Calibration Services

- **Distribute in North America:**
  - Flux-gate DCCTs from Danisense (Denmark)
  - Rogowski Coils from PEM (UK)
  - Passive CTs from MagneLab (US)
  - Particle Beam Diagnostics from Bergoz (France)
- **Manufacture & Distribute:**
  - Hall Effect-based Clip/Clamp-on Sensors
- **17025 Accredited Calibration Services**
  - DC up to 11 kA
  - AC up to 8 kA and up to 400 Hz
  - Onsite Services

GMW Current Calibration Lab

DANISENSE

PEM
Power Electronic Measurements

GMWAssociates

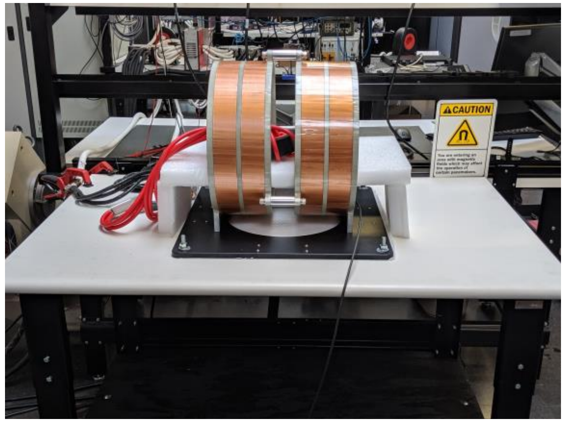
MAGNELAB

bergoz
INSTRUMENTATION

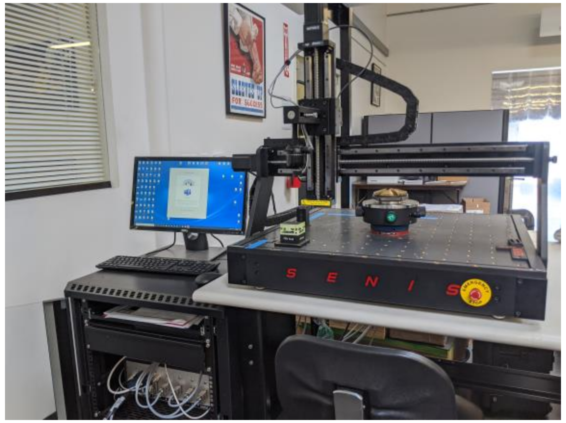
Magnetic Field Transducers & Calibration Services

- **Distribute in North America:**
  - Low-Field Flux-gate Magnetometers from Bartington (UK)
  - Hall Effect Digital Teslameters from Senis & Metrolab (Switzerland)
  - Analog and Digital Magnetic Field Transducers from Senis (Switzerland)
  - Hall Effect Magnetic Field Mappers from Senis & Metrolab (Switzerland)
  - NMR Teslameters & Magnetic Field Cameras from Metrolab (Switzerland)
- **Services:**
  - Magnetic field transducer calibration
  - Magnet mapping
  - Magnetic site survey
  - Magnetic field exposure testing

AC Coil @ GMW



Senis Mapper @ GMW



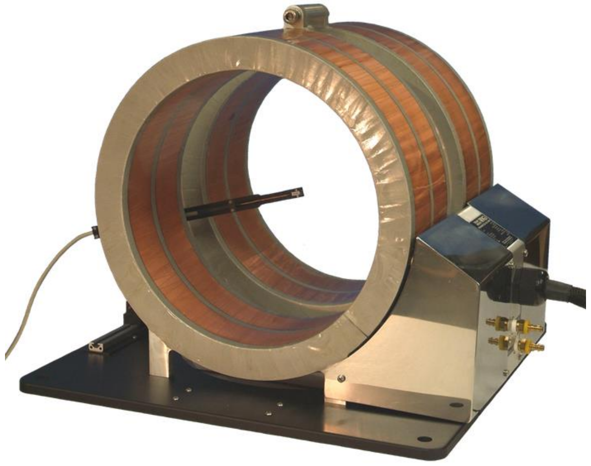
Bartington Instruments

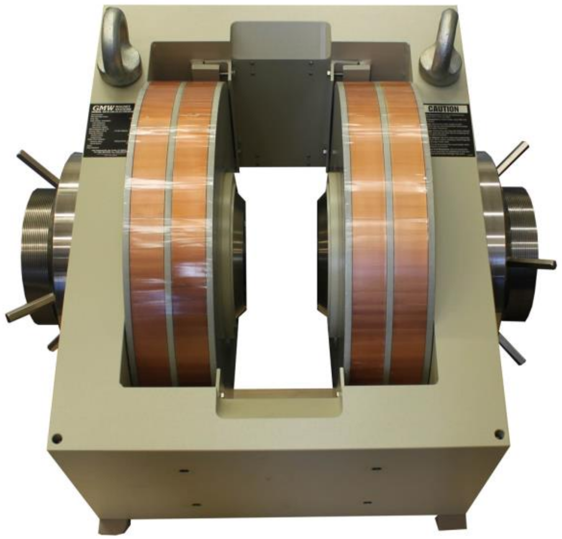
SENIS magnetic & current measurement

metrolab magnetic precision has a name

GMW Associates

Electromagnet Systems





GMW & Bartington Helmholtz Coils

- Environmental Field Exposure
- Magnetic Field Immunity Testing
- Shielded versions available

GMW Dipole Electromagnets

- Material Characterization
- Hall Effect Sensor Calibration

GMW Projected Field Electromagnets

- Sensor Testing

Current Transducer Technologies

GMW Associates

Current Shunts vs Galvanically Isolated Sensors

Current Shunts
Directly Measure Current

- Power Dissipation $\propto I_p^2$
- Inaccuracies due to heating at high I_p
- Limited dynamic range in high I_p shunts
- High Voltage Safety concerns

Galvanically Isolated Sensors
Measure Magnetic Field

Transformer-based:

- Passive Current Transformers (AC, highest hi-freq cut-off)
- Active Current Transformers (AC, highest sensitivity)
- Rogowski Coils (AC, moderate BW for very high currents)
- Closed-Loop Flux-Gate DCCTs (DC+AC, ppm-level accuracy)

Core-less Hall Effect-based

- Low-cost
- DC + AC up to 10s kHz
- ~1% accuracy
- Very high current

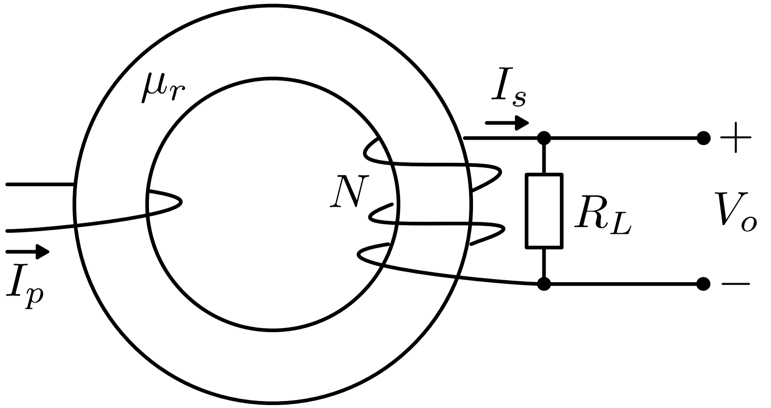
Passive Current Transformers (CTs)

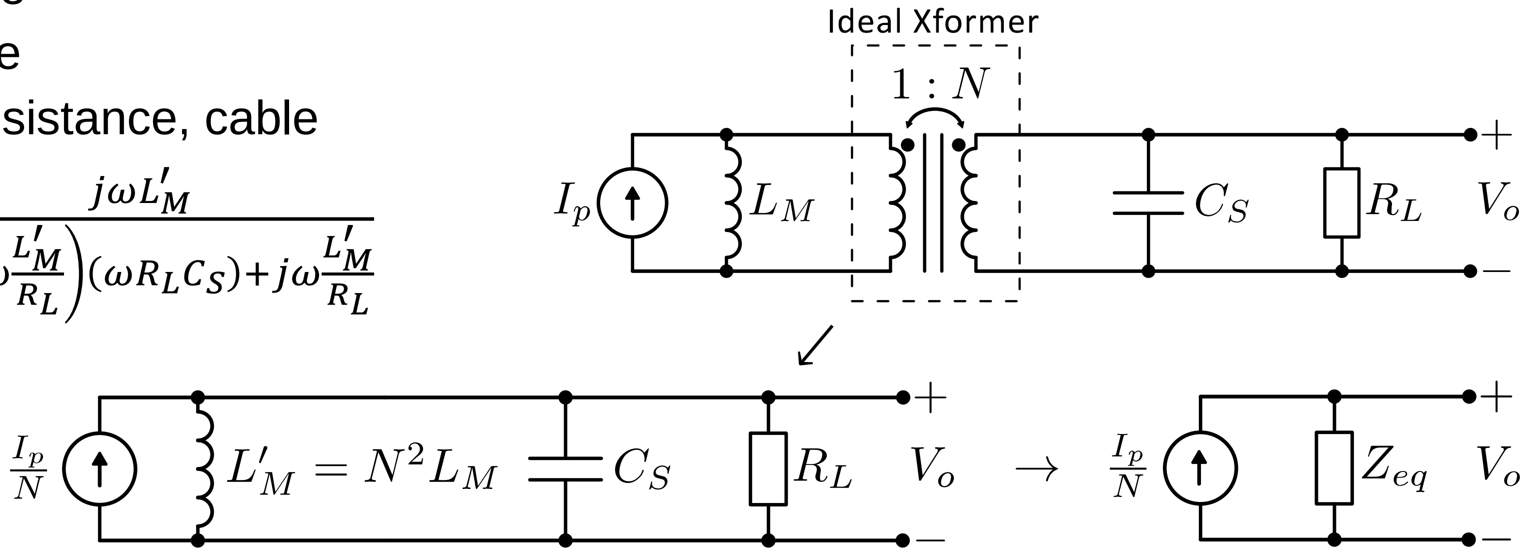
For AC & Pulsed Measurements

GMW Associates

Passive Current Transformer Model

- **Simplest** Galvanically isolated current sensor
- **AC Only - BW can exceed 1 GHz**
- Transformer with output $\mathrm{R_L} = 50\ \Omega$ termination resistor
- $V_o \cong \frac{R_L}{N} I_p$ within the pass band
- Circuit Model:
  - L_M: Primary magnetizing inductance
  - C_S: Secondary parasitic capacitance
  - Ignored leakage inductance, coil resistance, cable
- $\frac{1}{Z_{eq}} = \frac{1}{R_L} + \frac{1}{j\omega L'_M} + j\omega C_S \rightarrow Z_{eq} = \frac{j\omega L'_M}{1-\left(\omega\frac{L'_M}{R_L}\right)(\omega R_L C_S)+j\omega\frac{L'_M}{R_L}}$
- $\frac{V_o}{I_p} = \frac{Z_{eq}}{N} = \frac{1}{N}\frac{j\omega L'_M}{1-\left(\omega\frac{L'_M}{R_L}\right)(\omega R_L C_S)+j\omega\frac{L'_M}{R_L}}$
- **Band-pass filter**

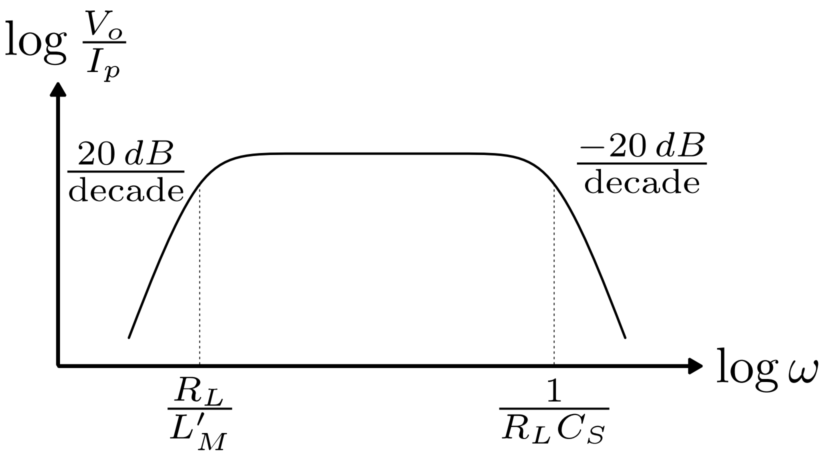




GMW Associates

Passive CT Frequency and Time Response

- $\frac{V_o}{I_p} = \frac{Z_{eq}}{N} = \frac{1}{N}\frac{j\omega L'_M}{1-\left(\omega\frac{L'_M}{R_L}\right)(\omega R_L C_S)+j\omega\frac{L'_M}{R_L}} = \begin{cases} \frac{j\omega L'_M}{N} & \omega \ll R_L/L'_M \\ \frac{1}{j\omega C_S N} & \omega \ll 1/R_L C_S \\ \frac{R_L}{N} & R_L/L'_M \ll \omega \ll 1/R_L C_S \end{cases}$

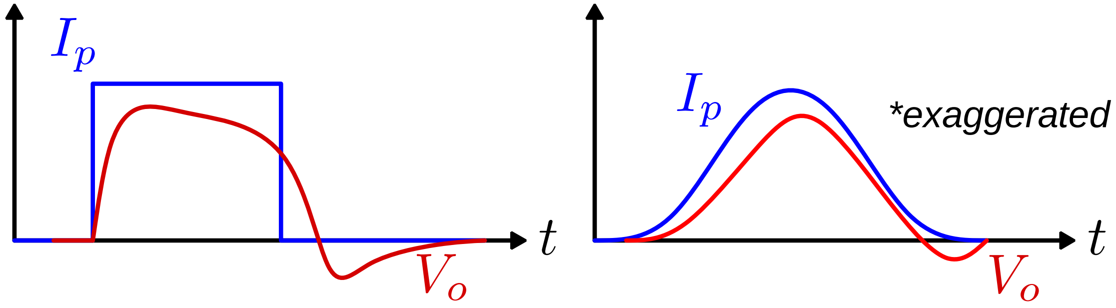


- **Low** frequency cutoff causes **droop** $t_d = \frac{\ln 0.9 - \ln 0.1}{\omega_L} \approx \frac{1}{3\,f_L}$ need to **maximize L'_M → high μ_r, N**
  - Measuring **low current** → high sensitivity → low N → low L'_M → **high f_L and significant droop**

- **High** frequency cutoff restricts **rise time** to $t_r = \frac{\ln 0.9 - \ln 0.1}{\omega_H} \approx \frac{1}{3\,f_H}$ need to **minimize C_S**
  - Measuring **high current** → low sensitivity → high N → high C_S → **low f_H and slow rise time**

- **R_L also affects BW**

- Waveforms get skewed:



GMW Associates

MagneLab CTs & Bergoz FCTs

MagneLab CT

- Sensitivity 0.025 to 2.5 V/A into 50 Ω
- Low Cutoff down to **0.5 Hz** (for low sensitivity)
- High Cutoff up to **500 MHz** (for high sensitivity)
- uA to 20 kA peak
- ID from 0.25 in to 2 in

Bergoz FCT

- Sensitivity 0.25 to 5 V/A into 50 Ω
- Low Cutoff down to **1.6 kHz**
- High Cutoff up to **1.5 GHz**
- Up to 2 kA peak
- ID from 22.2 mm to 198.4 mm

Passive CT Pitfalls

- **CT specs** typically guaranteed only for **50 Ohm termination**
- There is **finite insertion impedance** (order of 100 nH) that can **perturb DUT**
  - Added inductance can cause oscillations with package parasitics in SiC/GaN transistors switching
- **Noise** can get coupled through CT case, need **Common Mode Chokes** for noisy environments
- **Noise due to resistor** even though no active electronics (10 uVrms at 300 K over 500 MHz)
- **DC current** will cause the ω_L to go up, droop will increase
  - Susceptivity to external magnetic fields
  - Unipolar pulses may need a negative DC bias
- **I * t** needs to be **higher than primary pulse charge**

Lower cut-off frequency dependence to DC primary current

GMW Associates

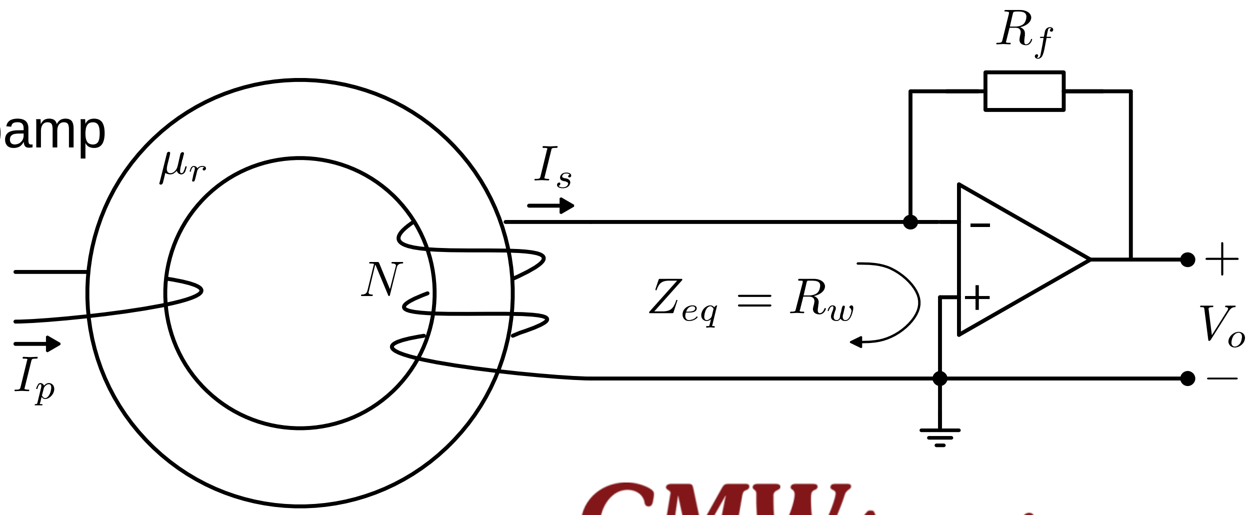
Active Current Transformers

For Low-Current AC Measurements

GMW Associates

Active (Hereward) Transformer

- **Passive CTs** are **limited**:
  - Cannot measure **long pulses** due to droop / ω_L
  - **Sensitivity limited** by minimum amount of turns
- In an **Active CT** coil load is only it's winding resistance R_w
- $\boldsymbol{\omega'_L = \frac{R_w}{L'_M} \ll \frac{50\ \Omega}{L'_M}}$
- ω_H additionally limited by Opamp finite Gain-BW
- $V_o = -\frac{I_p}{N} R_f$
- **Sensitivity can be high**, determined by R_f & opamp
- Another variation uses feedback coil



Bergoz ACCT

Bergoz ACCT

- Sensitivity 5 V/A to **10 kV/A** into high impedance
  - 3-range electronics as an option
- Low Cutoff < **3 Hz**
- High Cutoff up to **3 MHz**
- Full scale range **1 mA** to **2 A**
- ID from 22.2 mm to 198.4 mm

In-Flange for beam measurements

In-air

Single-range electronics

3-range electronics

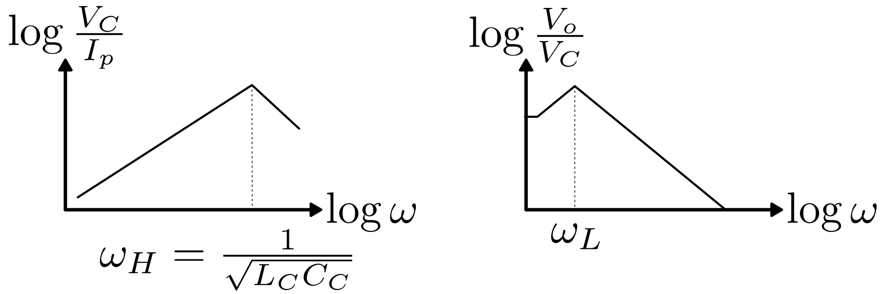
Rogowski Coils

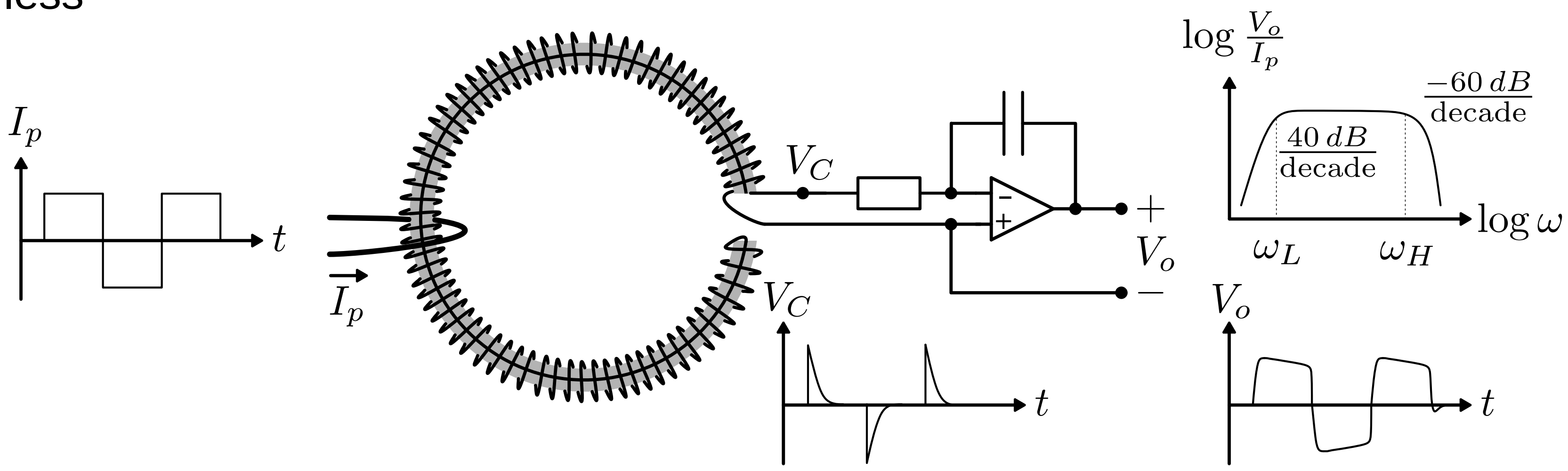
For High-Current AC Measurements

GMW Associates

Rogowski Coil Principle of Operation

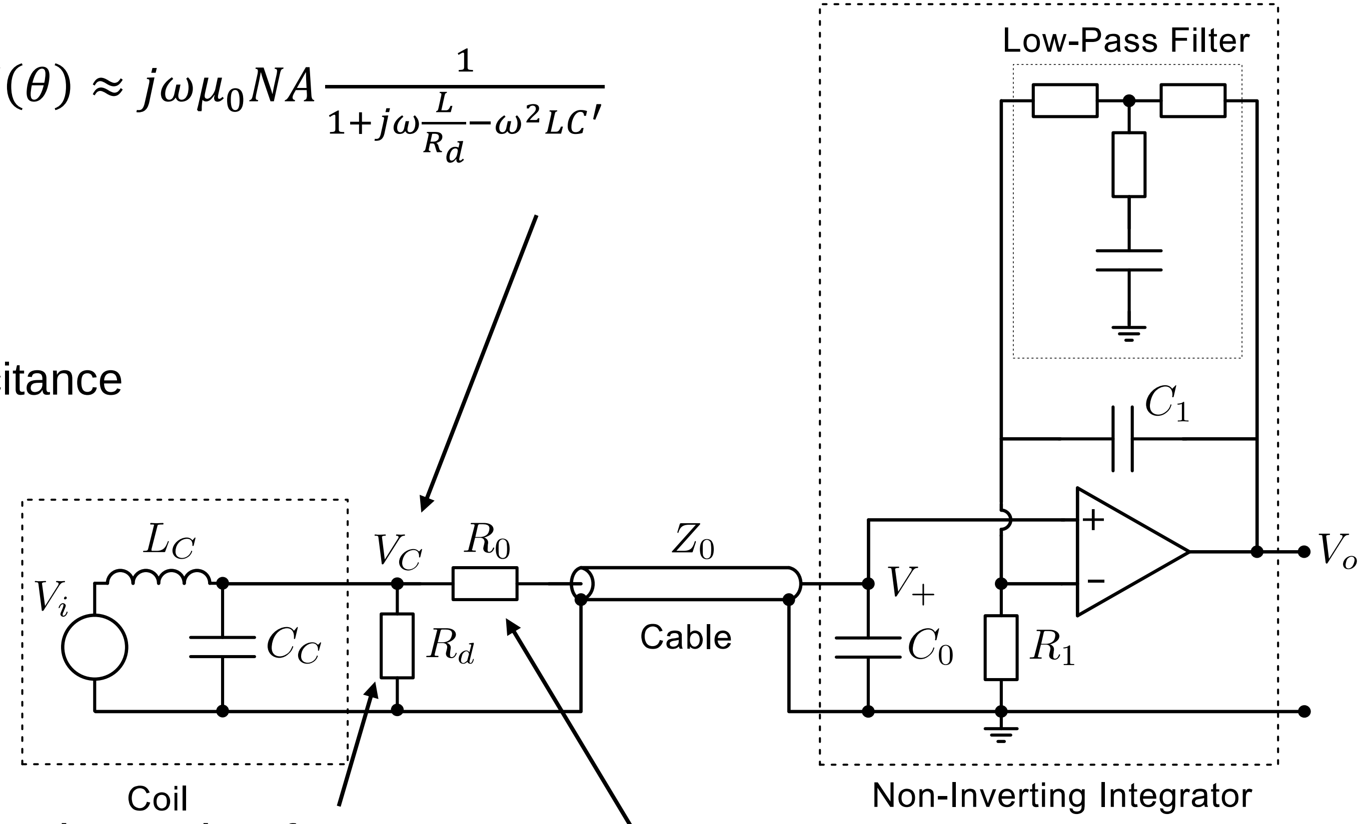
- Rogowski coil is a **Core-Less** Narrow-Band transformer operating before resonance
  - **Output** voltage is **derivative** of primary **current** – **need integrator** electronics
- Integrator results in **Band-Pass Filter**
  - **Low cut-off** causes **droop**
  - **High cut-off** limits **rise time**
- Can **measure** currents up to **100s of kA**
  - **No saturation** because core-less
  - **No overcurrent damage**
- **Limited by** $\left.\frac{dI_p}{dt}\right|_{pk}$ & $\left.\frac{dI_p}{dt}\right|_{rms}$





Rogowski Coil Circuit Analysis (PEM Architecture)

- Coil terminal transfer function $\frac{V_C}{I_p} = j\omega\mu_0 NA\, F(\theta) \approx j\omega\mu_0 NA \frac{1}{1+j\omega\frac{L}{R_d}-\omega^2 LC'}$
  - N is turns/m
  - A is former cross-section
  - $C' = \left(\frac{2}{\pi}\right)^2 C$
  - L and C are distributed inductance and capacitance
  - $F(\theta)$ is function of L & C
- Coil BW is $\omega_H = \frac{1}{\sqrt{LC'}}$

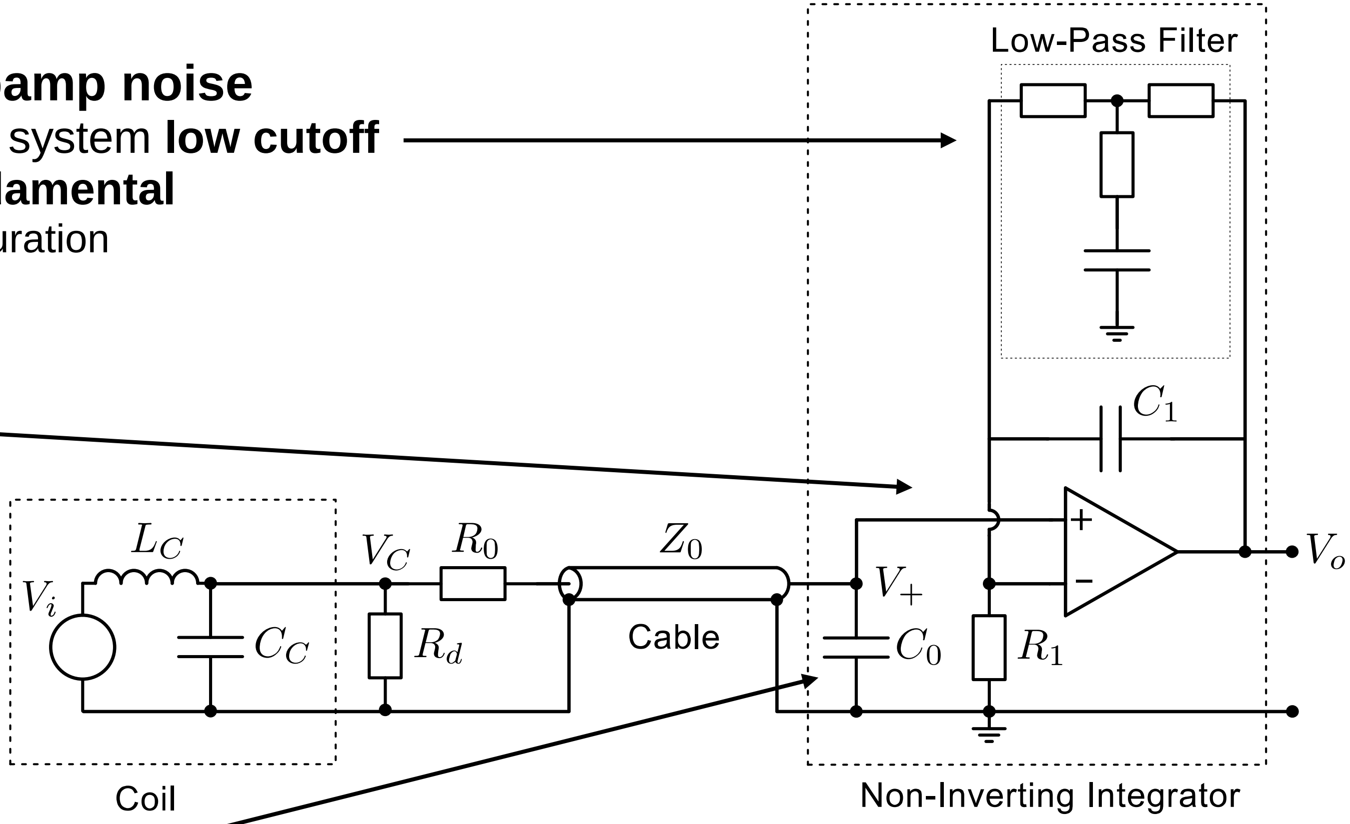


- Coil termination resistance R_d chosen for damping ratio of $\xi = 0.5$
- $R_0 \gg Z_0$ is typically fitted on coil end to *match* coil to the cable and forms part of the integrator

GMW Associates

Rogowski Coil Circuit Analysis Cont'd (PEM Architecture)

- **Low-pass** filter **attenuates** low-frequency **opamp noise**
  - **Trade-off** between **low frequency noise** and system **low cutoff**
  - **Tailored Narrow-Band Probe to reject Fundamental**
    - Cannot be done in Magnetic Cores due to Saturation
- At **low frequencies** $1/\omega C_0 \gg R_0$
  - $R_0 C_0$ has unity gain
  - Active integration with $R_1 C_1$
- At **high frequencies** $1/\omega C_1 \ll R_1$
  - Opamp has unity gain
  - Passive integration with $R_0 C_0$



GMW Associates

PEM Rogowski Coil Product Lines

CWT Ultra Mini
1.7 mm cross-section
1.2 kV isolation
30 A to 6 kA
2 Hz to 30 MHz

CWT Mini50HF
3.5 mm cross-section
2 kV isolation
30 A to 30 kA
1 Hz to **50 MHz**

CWT LF
Miniature & Standard Coils
60 A to 300 kA
8 mHz to 12 MHz

CWT
8.5 mm cross-section
10 kV isolation
30 A to 300 kA
0.03 Hz to 16 MHz

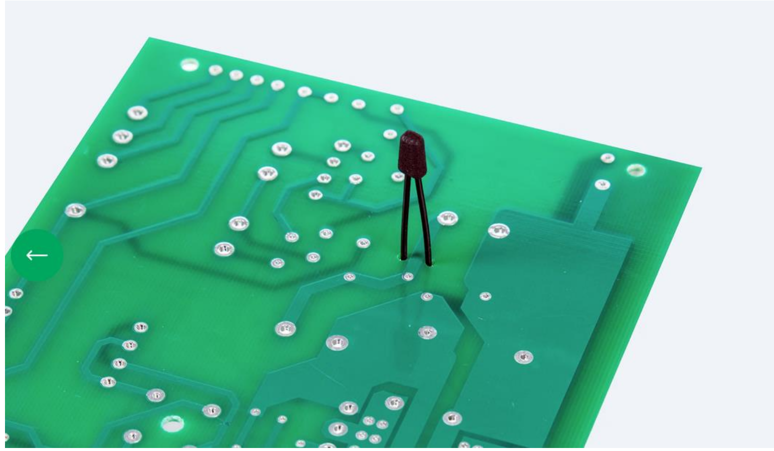
LFR
Dual Range
2 kV isolation
60 A to 60 kA
70 mHz to 1 MHz

RCTi & RCTi-3ph
For permanent installation
2 kV isolation
250 A to 50 kA
0.2 Hz to 1 MHz

PEM Rogowski Coils have a lot of Flexibility and can be Customized
Almost any coil length
Low Freq can be tailored

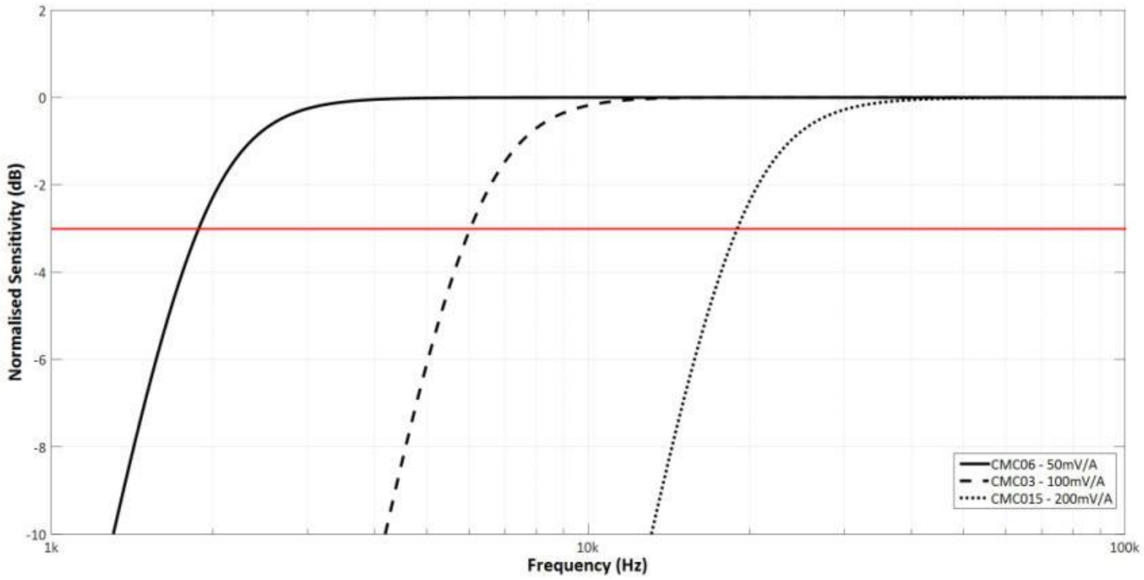
New “Forked” PEM Rogowski Coil

- **CWTUM-F / CTWUMHF-F**
  - 55 mm length
  - 1.2 mm & 1.7 mm (HF) cross-section
  - 1.2 kV isolation
  - 60 A to 12 kA
  - 1.2 Hz to 20 MHz and 30 MHz (HF)



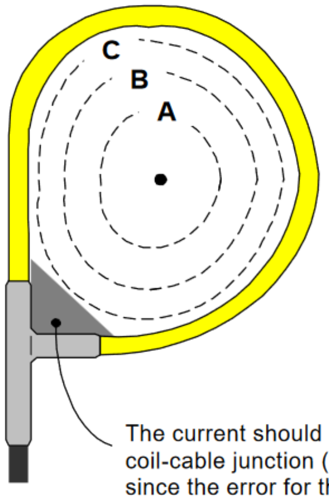
PEM Common Mode Current Rogowski Coil

- Probe for measuring **HF common mode current** in **VSDs**
- **High LF** bandwidth to **attenuate large fundamental** freq
- HF Bandwidth **10+ MHz** (up to 50 MHz)
- 37.5 A to 150 A



Rogowski Coil Pitfalls

- **AC-only**
- **Damage** can occur due to **excessive** $\left.\frac{dI_p}{dt}\right|_{pk}$ **and/or** $\left.\frac{dI_p}{dt}\right|_{rms}$ – not absolute current I_p
- For **high** $\frac{dV}{dt}$ applications there are version with **electrostatic screen** around coil to reduce noise
- Primary current **cable** should **not** be position **close to the coil-cable junction**
- **Very thin** Rogowski Coils (Ultra Mini) **need care** to **avoid damage** (no sharp bends or edges)
- The **longer** the coil, the **lower** ω_H
- The **lower current rating**, the **higher** ω_L



POSITIONAL ACCURACY OF A STANDARD ROGOWKSI COIL -
% error with a point source of current

Type	A	B	C
Miniature Coil	±0.5%	±1%	±3%
Standard Coil	±0.5%	±1%	±2%

The current should not be positioned close to the coil-cable junction (shown by the shaded area) since the error for this region is greater.

GMW Associates

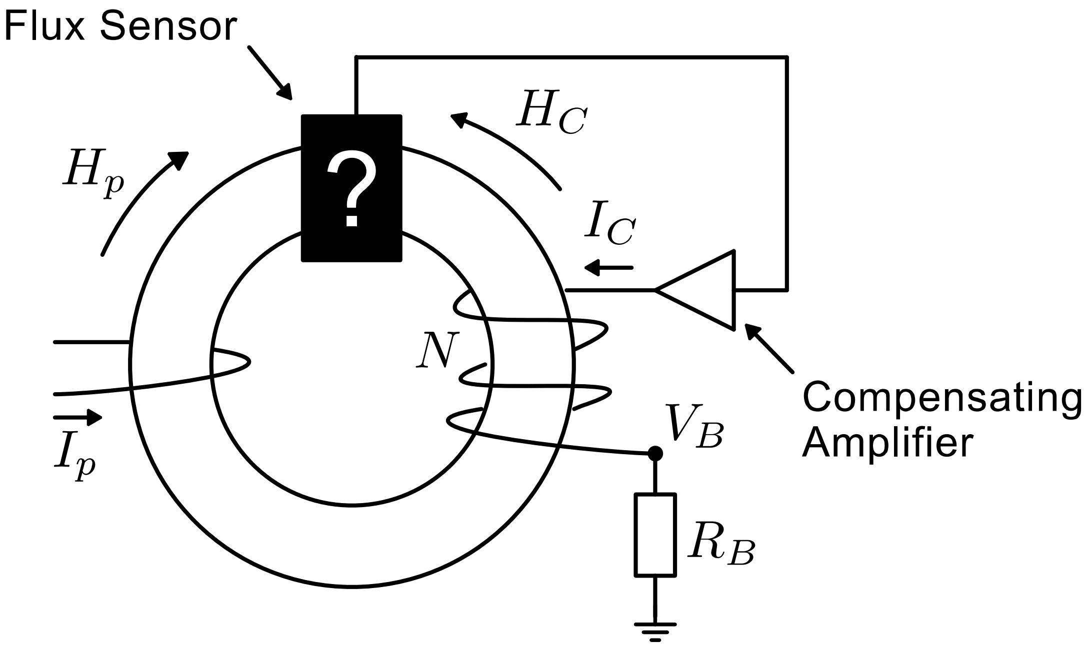
Closed-Loop Flux-Gate DCCTs

For **High-Precision** DC & AC Measurements

GMW Associates

Closed-Loop or Zero-Flux Concept

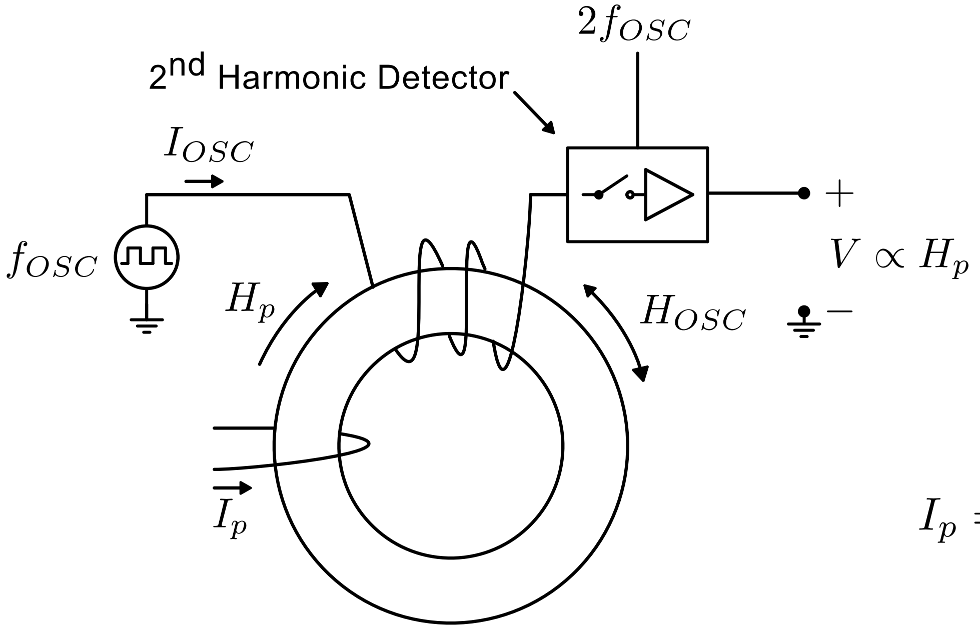
- **Flux Sensor** senses Flux in Core
  - **Hall-Effect** Sensor in Gap (Low-precision due to gap)
    - Temperature changes causes Mechanical changes
    - Sensitivity to external fields due to gap
  - **Coil** (Hereward Transformer, AC Only)
  - **Flux-gate** (most precise)
- **Feedback loop** maintains **Zero-Flux** in Core
  - Generates opposing current I_C into N turns
- I_p measured through **secondary current** $I_C = \frac{I_p}{N}$
- I_C measured **directly** or through **burden resistor** R_B
  - Typically with DMM or Power Analyzer

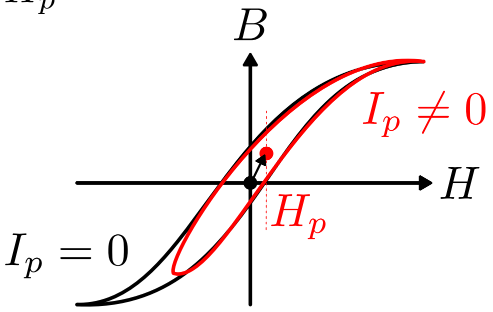


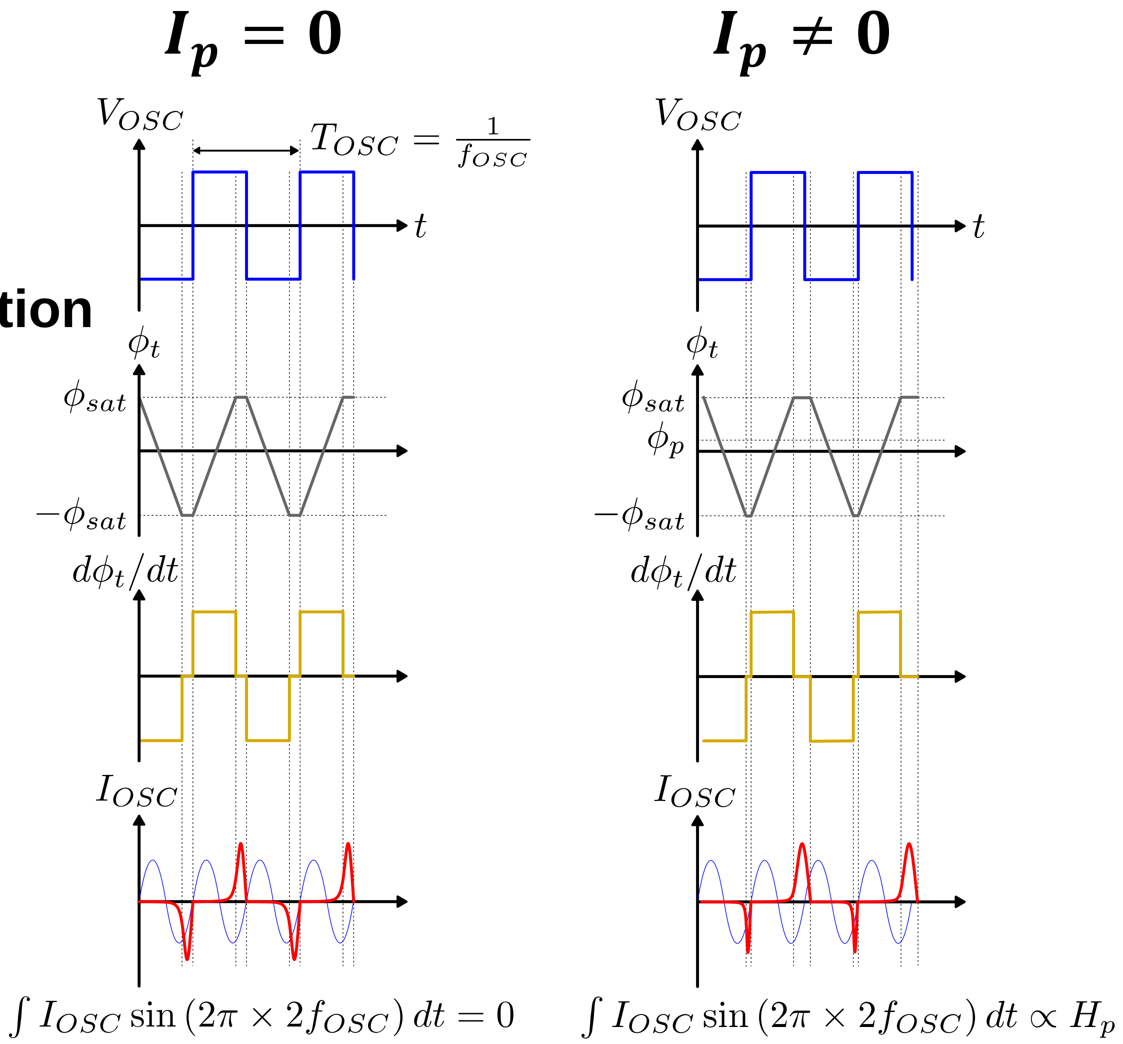
GMW Associates

Flux-Gate Concept

- Coil $\mathcal{EMF} = -NA\frac{d}{dt}[\mu(t)H(t)]$
- If $\frac{d}{dt}H(t) = 0$ then $\mathcal{EMF} \neq 0$ only for $\frac{d}{dt}\mu(t) \neq 0$
- **Modulate $\mu(t)$ by cycling core in and out of saturation**
- Finite I_p results in **2nd Harmonic Signal $\propto I_p$**

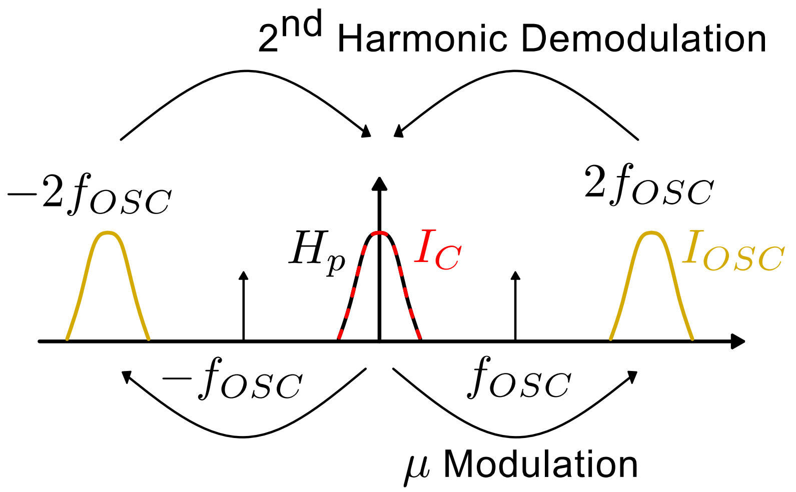


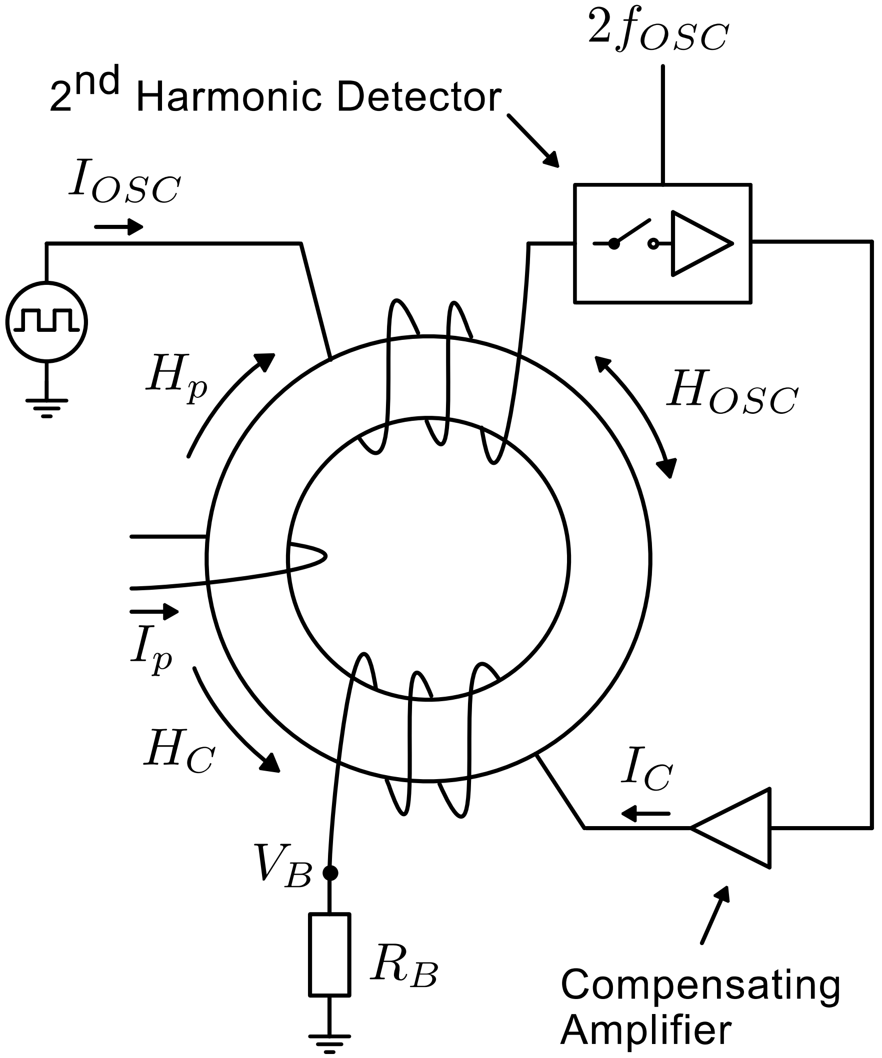




Closed-Loop Flux-Gate

- **Modulating** core $\boldsymbol{\mu}$ generates **2nd harmonic** $\propto \boldsymbol{I_p}$
- Demodulated 2nd Harmonic **drives compensation loop**
- I_p measured through **secondary current** $I_C = \frac{I_p}{N}$
- I_C measured **directly** or through **burden resistor** R_B
- **Frequency Mixing Process** due to **Core Non-Linearity**:
  - Modulating core μ with f_{OSC} **up-mixes** H_p spectrum to 2 f_{OSC}
  - 2nd Harmonic detector **down-mixes** I_C to baseband
- *This does not work as is:*
  - Temperature drifts
  - Excitation breakthrough

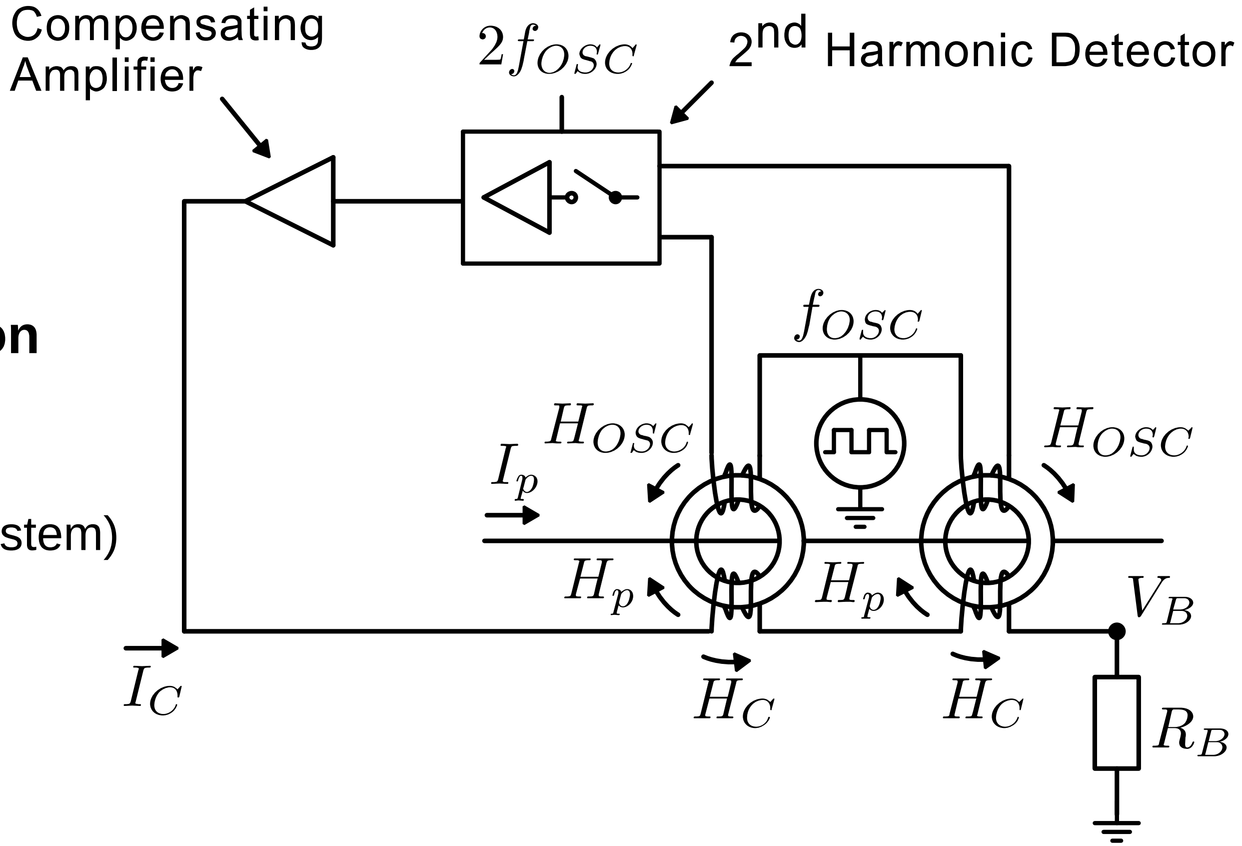




GMW Associates

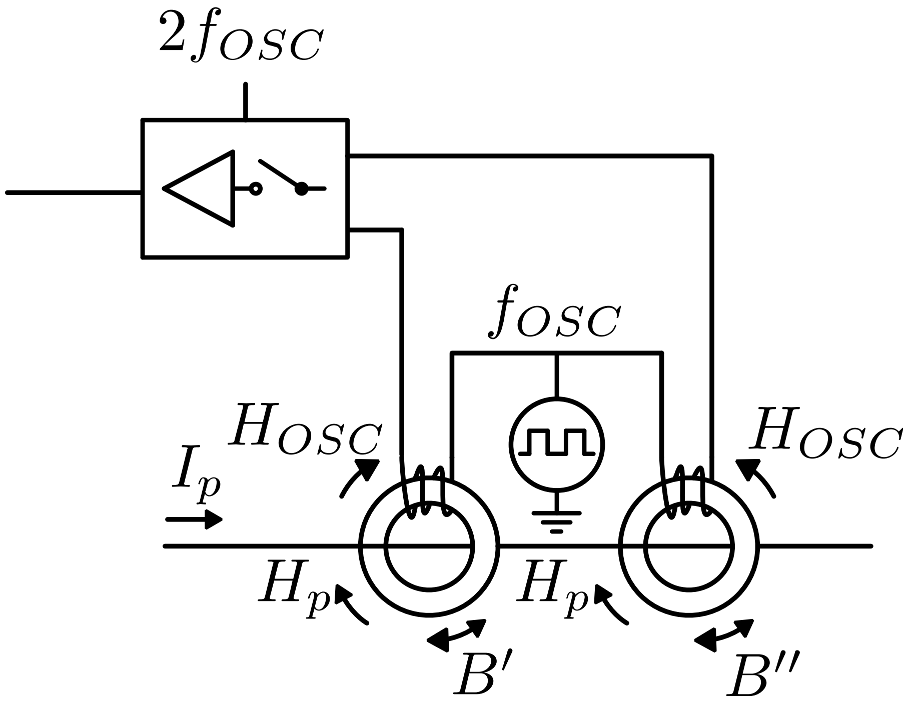
2-Core Flux-gates

- **Single-core** flux-gates suffer from
  - **Temperature drifts**
  - **Excitation breakthrough** to the output
- **2 matched cores** to **cancel drifts** and **excitation**
  - Cores Modulated in opposition
  - Demodulated 2nd harmonic difference drives I_C
  - **Reduced cross-talk** to other sensor (e.g. 3ph system)
- Many different excitation / demodulation circuits



Mathematical Analysis of the Flux-Gate

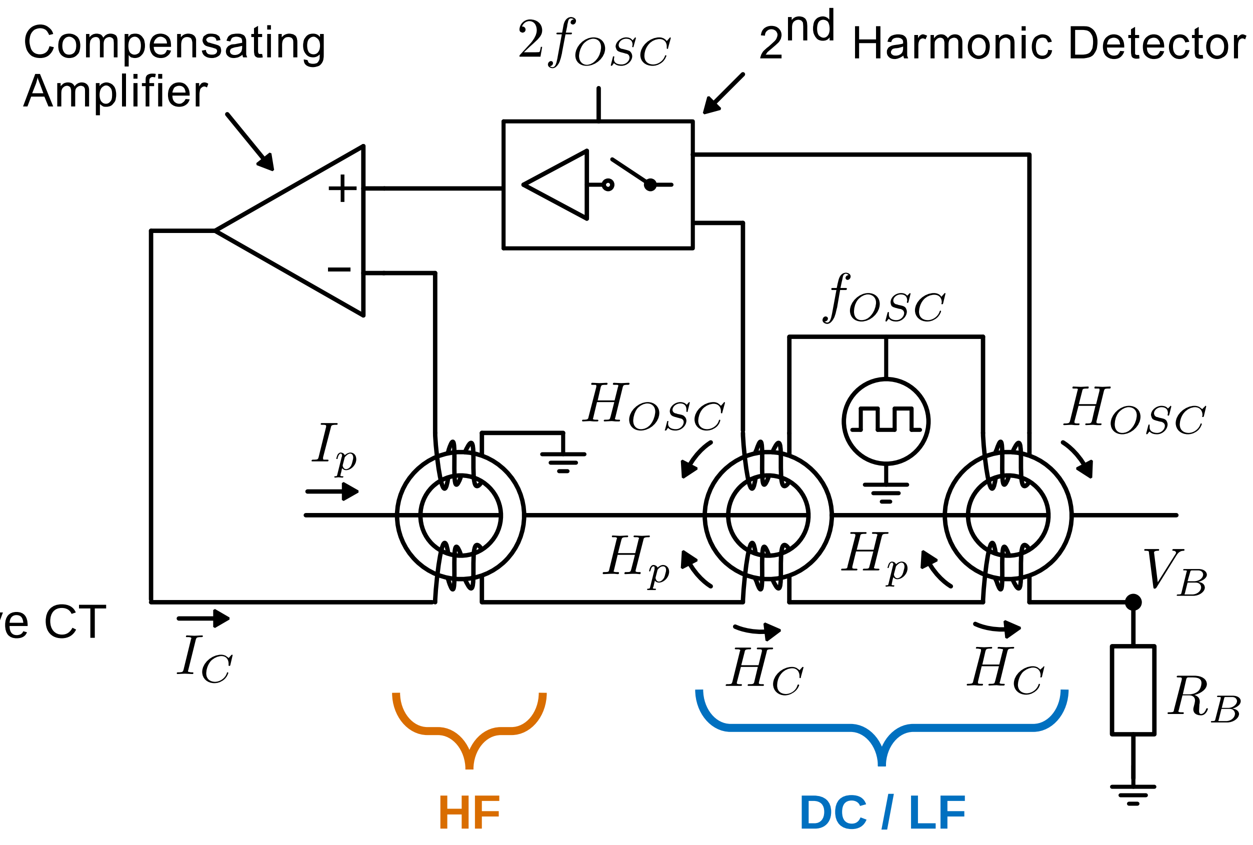
- B-field in the two cores: $\left| \begin{array}{l} B' = f(H_p + H_{OSC}) \\ B'' = f(H_p - H_{OSC}) \end{array} \right.$
- Sense coil $\mathcal{EMF} = -NA\frac{d}{dt}(B' + B'')$
- If $\boldsymbol{B} = \boldsymbol{aH}$, then $B' + B'' = 2aH_p$, and $\mathcal{EMF} = -2aNA\frac{d}{dt}H_p = 0$
  - **Flux-gates depend on the core non-linearity**
- If $\boldsymbol{B} = \boldsymbol{aH} - \boldsymbol{bH^3}$, then $B' + B'' = 2aH_p - 2bH_p^3 - 6bH_pH_{OSC}^2$
  - $\mathcal{EMF} = 6bNAH_p\frac{d}{dt}H_{OSC}^2 \neq 0$
  - If $H_{OSC} \propto \sin\omega t$ then $\mathcal{EMF} \propto \sin 2\omega t$
  - Note this is a simplified core model for qualitative analysis



GMW Associates

Extending the Frequency Response

- Flux-Gate **BW** on the **order of $\mathbf{100\ Hz\ -1\ kHz}$**
- **3rd Core** in **AC CT** configuration up to **MHz**
- Compensating Amplifier **sums LF and HF**
- Four frequency regions:
  - **Near DC: Flux-gate** and **Amp** drives I_C
  - **Low-Freq: Amp** sums **Flux-Gate + ACCT** drive I_C
  - **Medium-Freq: Amp + ACCT** drive I_C
  - **High-Freq:** Amp Unity Gain – **ACCT +** $\boldsymbol{R_B}$ as Passive CT



Danisense Closed-Loop Flux-Gate DCCTs

Product Range Overview

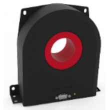
Output Type	Product Family	Primary Current (Arms)											
		50	100	200	300	400	500	600	1000	1200	2000	5000	10000
Current	DP	■	PCB Mount, Programmable, 12.5/25/50Arms										
	DT	■	■	■									
	DS	■		■	■	■		■					
	DQ						■	■					
	DC			■									
	DM									■			
	DL										■		
	DR											■	■
Voltage	DS	■		■	■	■		■	■				
	DW						■						
	DM									■			
	DL										■		
	DR											■	■


DP series


DS series


DQ series


DT series

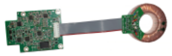

DM series



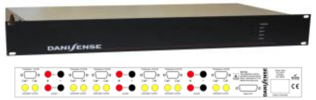
DR series


DW series


DL series

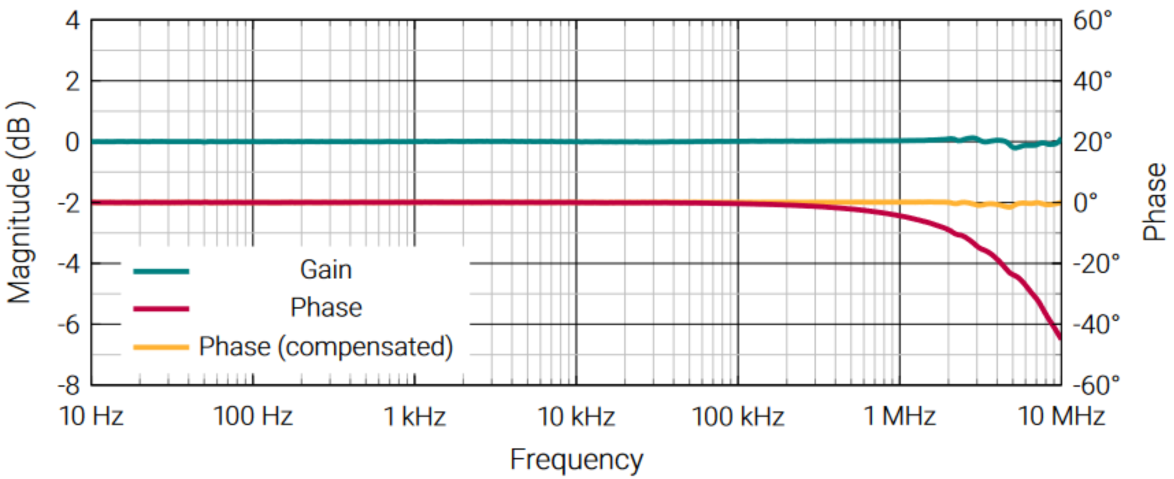

Unpackaged


DC series


DSSIU System Interface

Danisense DW500UB-10V – Highest Frequency DCCT

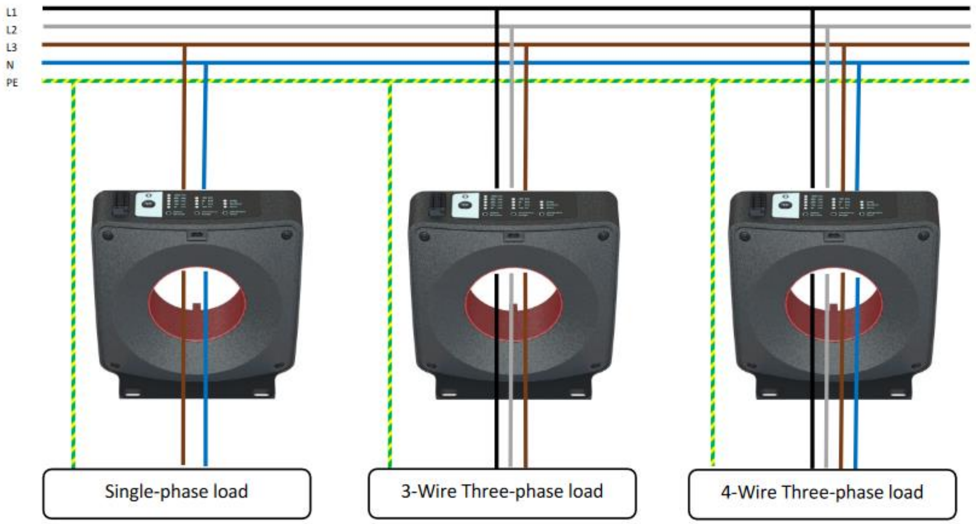
- Highest BW Closed-Loop Flux-Gate **DC – 10 MHz** (3dB)
- Up to **500 A** DC/AC
- 2 V output (250 A/V ratio)

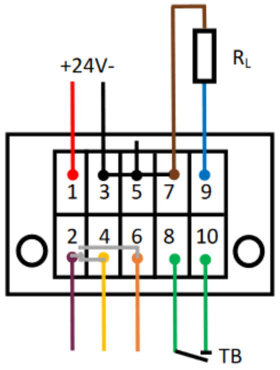


GMW Associates

Danisense Residual Current Monitor

- B/B+ Residual Current Monitor
  - **0 – 2 Arms**
  - **DC to 100 kHz**
- **4 – 20 mA** output for monitoring with PLC
- **Relay output** can trip breaker or contactor
- Model with **USB** for PC control and data logging



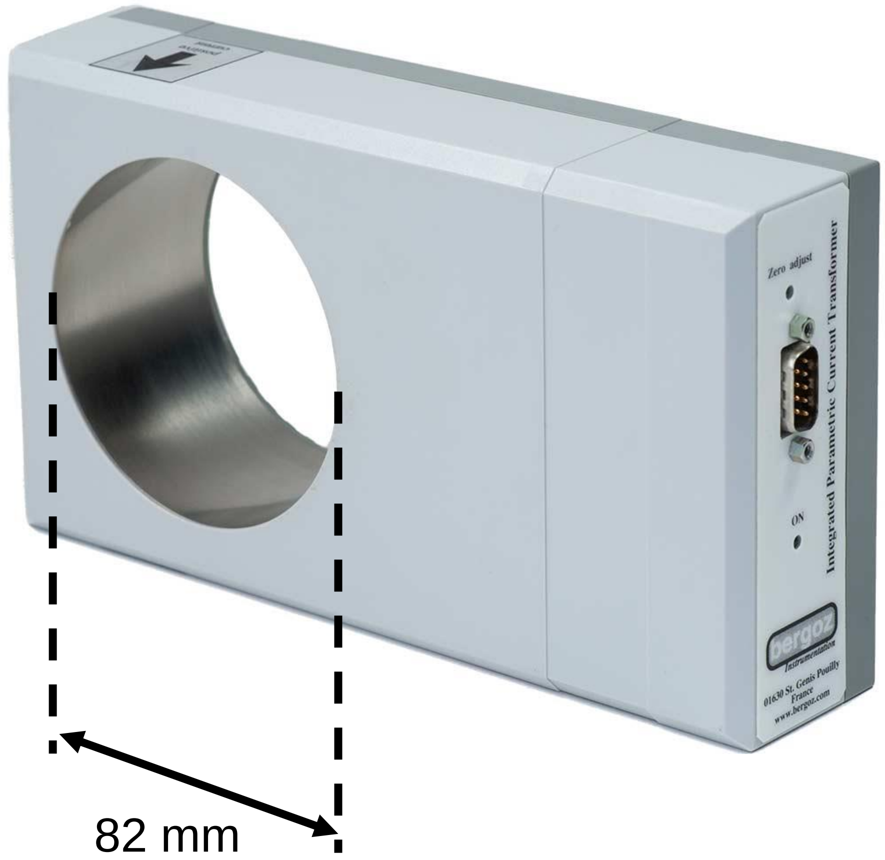


Pinout:

1: 24Vdc
2: Relay common
3: 0Vdc
4: Relay NC contact
5: 0Vdc
6: Relay NO contact
7: 0Vdc
8: External Test button, contact 1
9: Analog 4-20mA output
10: External Test button, contact 2

Bergoz IPCT – High Resolution DCCT

- Custom Full-scale from **1 mA to 20 A**
- **DC to 3.8 kHz** (3dB)
- 10 V output
- Zero-offset potentiometer
- **82 mm Aperture** accommodates Large Connectors or use in differential mode for residual current (e.g. x-ray tube)



Closed-Loop Flux-Gate DCCT Pitfalls

- **Opening** the DCCT **Secondary** can lead to transducer **damage**.
  - Danisense DCCTs have internal protection but many other DCCTs in the market do not.
- **Persistent over-current** will cause **damage**.
- An **over-current spike** will cause **oscillations** that dampen within milliseconds.
- **Turn-on history** has a small effect due to core hysteresis causing **small zero-offset drifts**.
- **Excitation frequency** will have a **small breakthrough** signal to the **output**.
- **Compensation current** comes from Transducer **power supply**, make sure it has enough **oomph**
- **Susceptivity** to **external** magnetic **fields**
  - Flux-gate DCCTs are less susceptible than Closed-Loop Hall Effect with gap but not completely immune

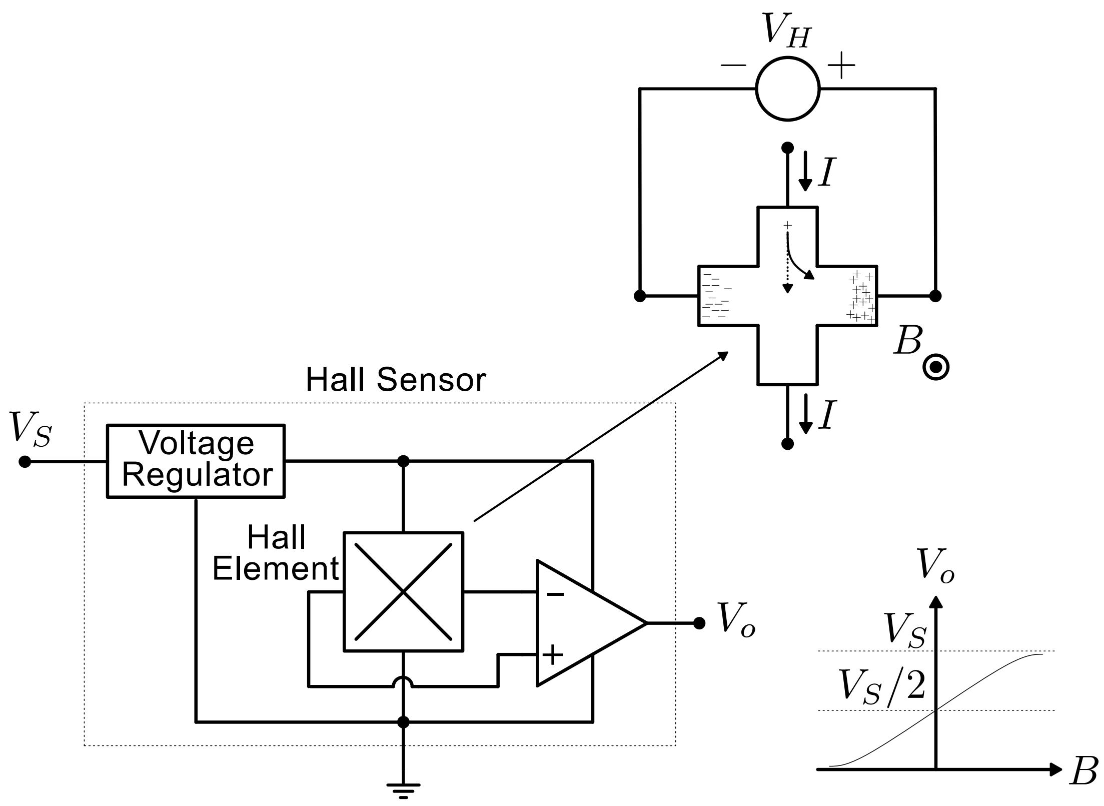
Hall Effect Sensors

For DC & AC Measurements

GMW Associates

Hall Effect Sensors

- **Hall Effect** is production of **Voltage** across conductor in **magnetic field transverse to current**
  - Arises from **Lorentz Force** on charge carriers
- **Hall Element:**
  - Typically p-type semiconductor
  - $V_H = R_H \left(\frac{\boldsymbol{I}}{t} \times \boldsymbol{B}\right)$
  - R_H is Hall Effect coefficient (material-dependent)
  - t is material thickness
- **Hall Sensor:**
  - Integrated Hall Element + Voltage Regulator + Amplifier
  - Output saturates near supply voltage and ground

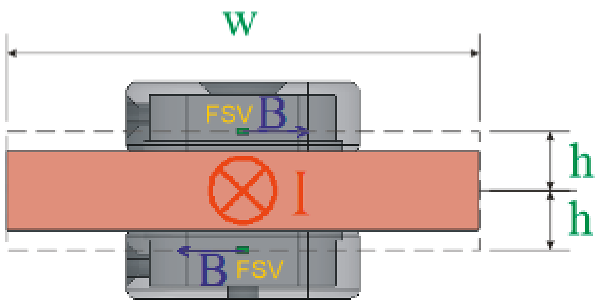


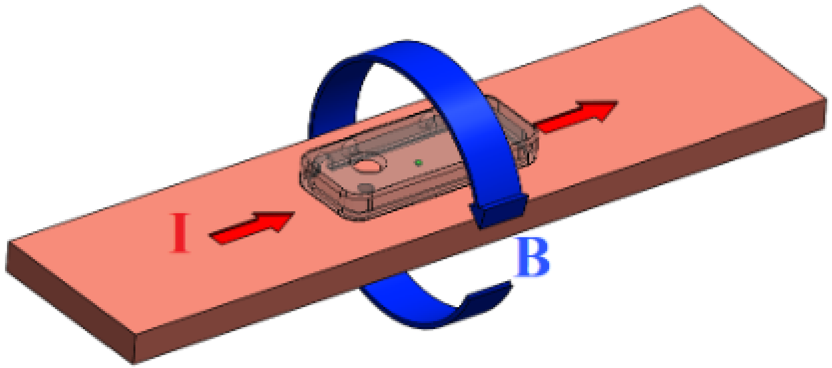
GMW Associates

Senis BBM

- Two interconnected Hall Sensors as a Busbar current transducer
  - Sensitivity depends on Busbar geometry but output needs calibration
  - External currents mostly rejected depending on orientation
  - Needs engineering effort from user
- $5\ V$ supply
- $0 \pm 4\ V$ output
- DC only (AC affected by busbar geometry)
- Clean recovery from overload

$$H(I, w, h) \approx \frac{I}{2(w + 2h)}$$





GMW CPC/CPCO

GMW CPC
250 A to 2 kA
1% accuracy
27 mm aperture
-40 degC to 100 degC
DC to 75 kHz
5 V supply
0 ± 2 V output

25 A in development

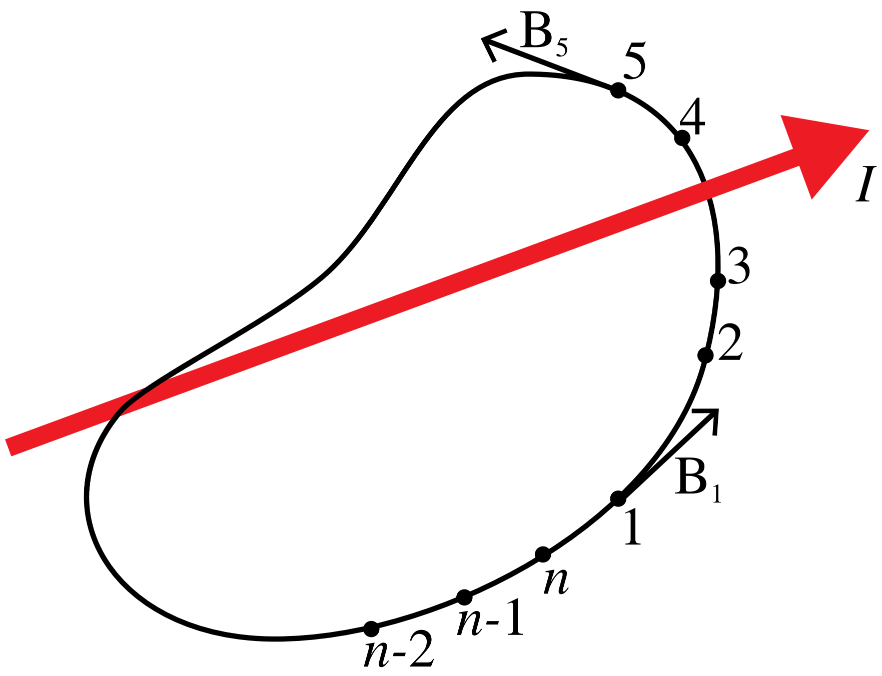
GMW CPCO
500 A to 16 kA
1% accuracy
77 or 160 mm aperture
-40 degC to 100 degC
DC to 40 kHz
5 V supply
5 ± 5 V, 0 ± 5 V, 0 ± 10 V, RMS
$0 - 3$ V, $4 - 20$ mA output

20 A (77mm) in development

GMW CSS-SO
400 A to 12 kA
1% accuracy
102mm x 30.2 mm aperture
DC to 1 kHz
5 V supply
0 ± 2 V output

GMW CPC/CPCO Principle of Operation

- Ampere's Law: $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I_{enc}$
  - Independent of position of the current
- Discretize Ampere's Law $I_{enc} = \frac{1}{\mu_0} \oint \boldsymbol{B} \cdot d\boldsymbol{l} \approx \frac{1}{\mu_0} \sum_{i=1}^{n} C_i B_i$
  - B_i is tangential field component at point i
  - C_i are constants determined by magnetic modeling
- Currents external to the integration path are rejected
  - **More sensors** result in **better rejection**



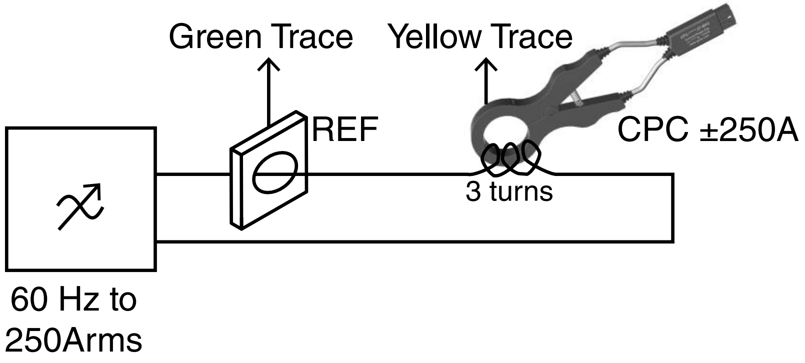
US Patents: 9952257, 10690701
European Patent: 2972425

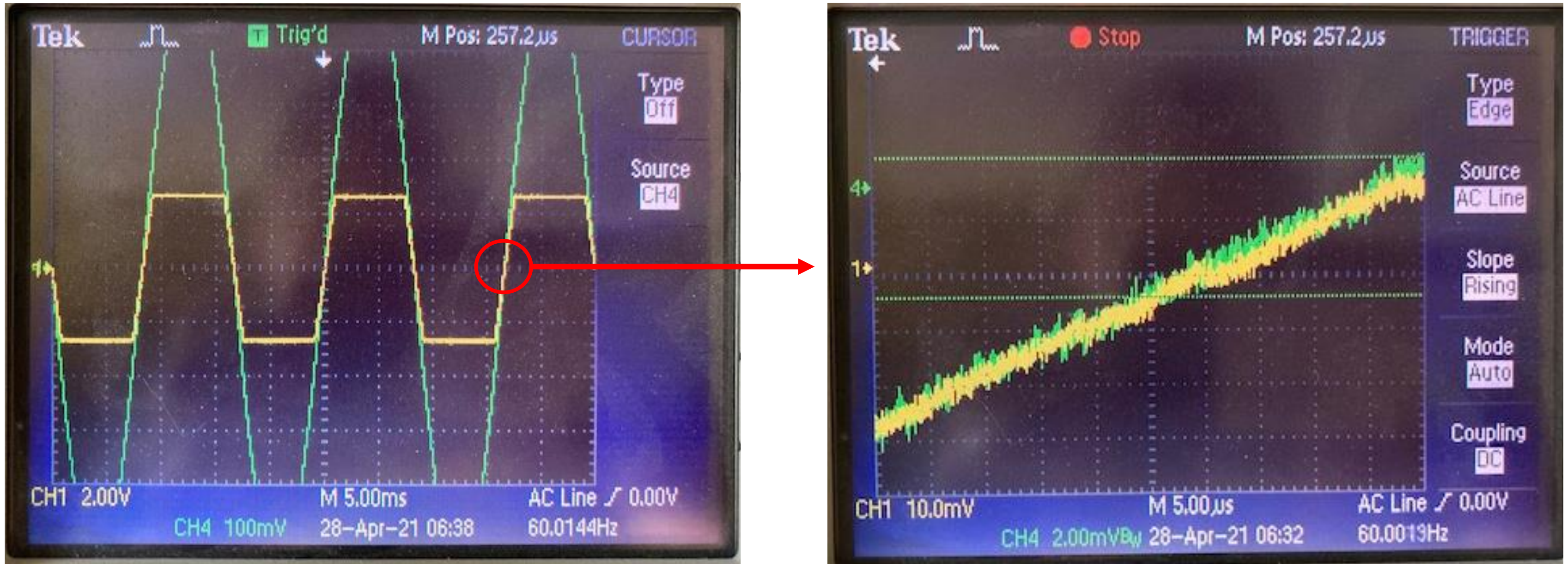
GMW Associates

Recovery from Overload

With 4x primary overload current the CPC shows:

- No electrical saturation, correct sign, no overshoot
- No ringing
- No zero-crossing phase shift after overload
- No damage





CPC ±250A fs with 750Arms Primary Current (yellow trace), or ±1060A. Approx. 4x full scale.

GMW Associates

Common Issues

GMW Associates

Transducer Termination

For Voltage-Output Transducers

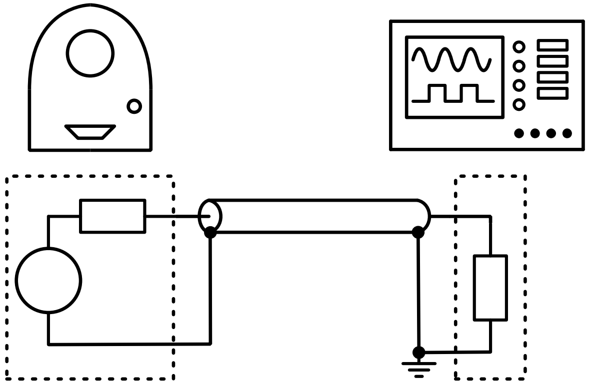
GMW Associates

Problem Overview

- Often asked about:
  - **Factor of 2 discrepancies** in measurements due to improper termination
  - Coaxial **cable reflections** when terminating into High-Impedance
  - There is often confusion between **Source** and **Termination Impedance**

- **Always Check the Manual for the Proper Termination Impedance!**

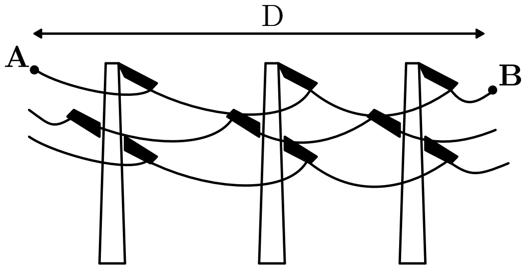
- We need to analyze the signal propagation in a coaxial cable terminated into $50\ \Omega$ and $1\ \mathrm{M}\Omega$
  - Only applicable to **Voltage-Output** transducers

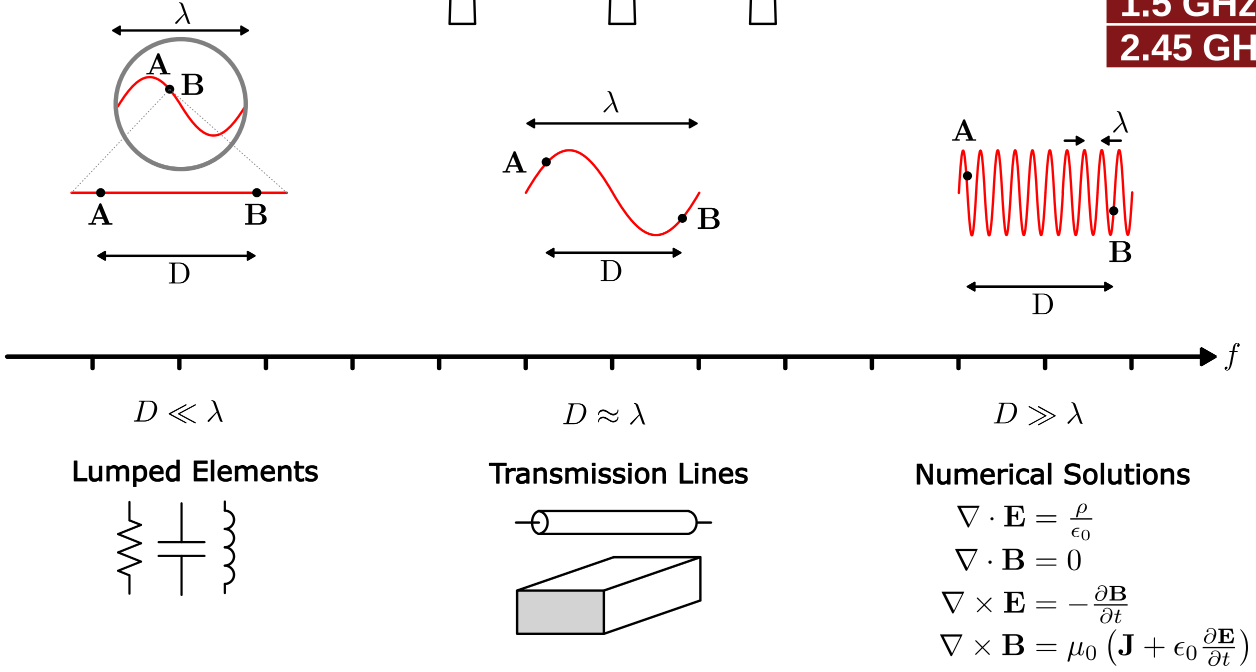


GMW Associates

Electrical Length

Frequency	Free-Space λ_0	Notes
60 Hz	5,000 km	AC power distribution freq in North America.
1 kHz	300 km	
75 kHz	4 km	Typical power converter switching frequency.
1 MHz	300 m	
10 MHz	30 m	Highest frequency **Danisense** transducer.
50 MHz	6 m	Highest frequency **PEM** Rogowski Coil.
500 MHz	60 cm	Highest frequency **MagneLab** CT.
1.5 GHz	20 cm	Highest frequency **Bergoz** FCT.
2.45 GHz	12 cm	Typical microwave oven magnetron frequency.





Wavelength refers to the highest frequency of interest

Methods of Analysis

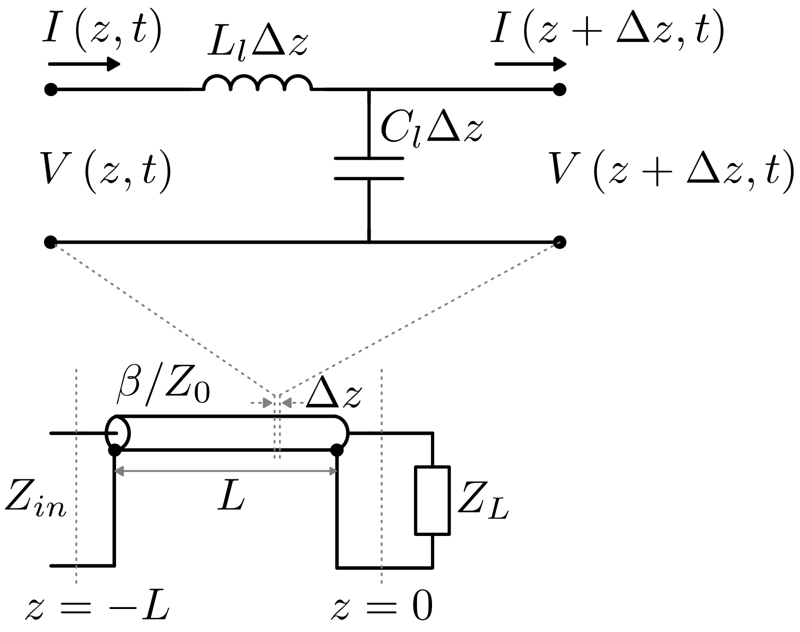
- **Lumped Element or Circuit Analysis**
  - **Mathematical approximation** to Maxwell's equations
  - Problem can be decomposed into **ideal circuit elements** (resistors, capacitors, inductors)
  - **Ignore finite speed of light**

- **Transmission Line Theory**
  - **Another Mathematical approximation** to Maxwell's equations
  - Problem can be decomposed into **transmission lines & ideal circuit elements**
  - **EM waves** propagate in **Transmission Lines** with finite speed
  - Any **junction** can cause **reflections**
  - Yields same results as Circuit Analysis in electrically small problems

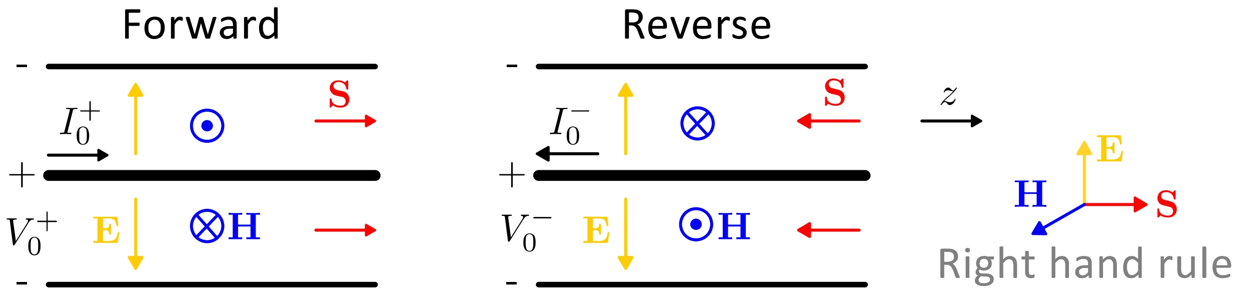
Transmission Line Theory

- **Transmission Line:** physical structure that guides EM waves without reflection or mode conversion (e.g. coax, rect waveguide)

$$V(z,t) = V_0^+ e^{j(2\pi f t - \beta z)} + V_0^- e^{j(2\pi f t + \beta z)}$$
$$I(z,t) = I_0^+ e^{j(2\pi f t - \beta z)} - I_0^- e^{j(2\pi f t + \beta z)}$$



- Transmission Line Properties:
  - Wave impedance $Z_0 = \frac{V_0^\pm}{I_0^\pm} = \sqrt{\frac{L_l}{C_l}}$
  - Propagation constant $\beta = \frac{2\pi}{\lambda_g}$, where λ_g is guided wavelength
- Minus sign in front of I_0^- sets power flow direction $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$



GMW Associates

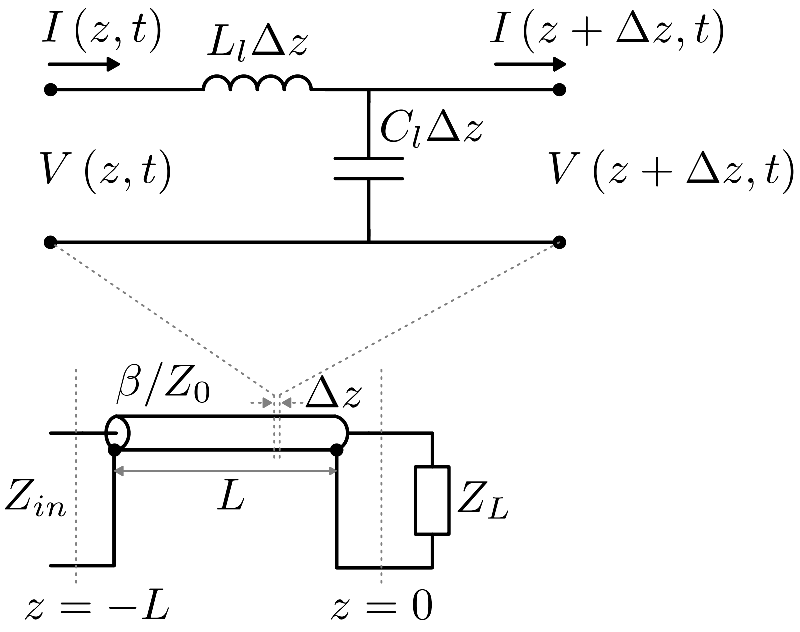
Reflections In Transmission Lines

- Terminating into $Z_L \neq Z_0$ causes reflection $\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$

- Voltage across the line is function of position (standing wave)
$$V(z) = V_0^+ e^{-j\beta z}\left(1 + \Gamma_L e^{j2\beta z}\right)$$

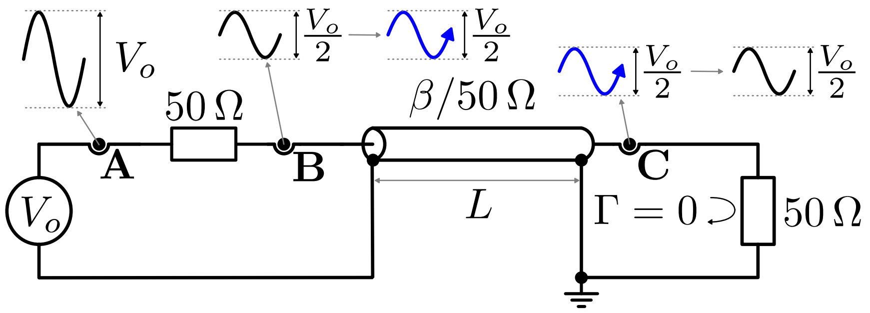
- Impedance from the input side becomes a function of the length
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)}$$

- Electrically short transmission lines are transparent $\lim_{\beta L \to 0} Z_{in} = Z_L$

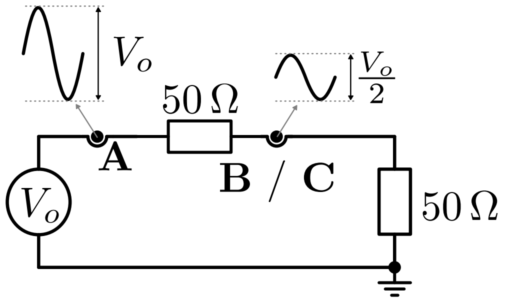


Terminating into 50 Ω

- **Transmission Line Analysis:**
  - Impedance seen from point B is 50 Ω
  - Voltage divider at point B launches wave with half amplitude
  - Wave fully absorbed at load
  - **Measured voltage is half**

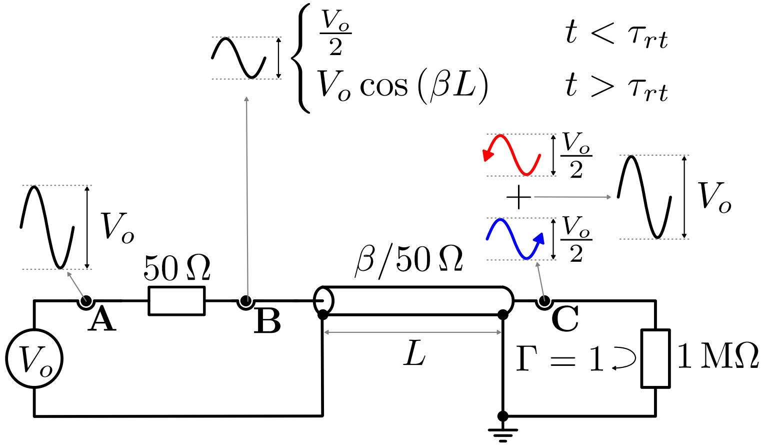


- **Circuit Analysis:**
  - Coax replaced by short-circuit
  - **Measured voltage is half** because of voltage divider

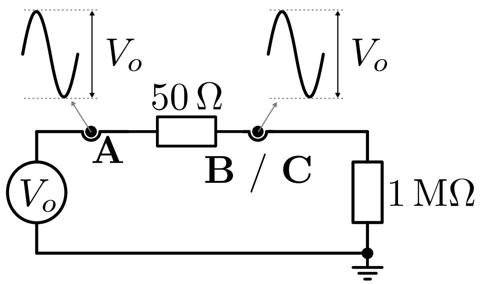


Terminating into High-Impedance

- **Transmission Line Analysis:**
  - Voltage divider at point B launches wave with half amplitude (initially)
  - Wave fully reflected at load
  - Voltage at point C is sum of Forward and Reverse wave amplitudes
  - **Measured voltage is full**
  - Reflected wave cause standing wave but fully absorbed at source



- **Circuit Analysis:**
  - Coax replaced by short-circuit
  - **Measured voltage is full** because of voltage divider



Miscellaneous Considerations

- Oscilloscope Termination Impedance:
  - Mid & Low-end scopes only have high-impedance input – for $50\ \Omega$ need BNC feedthrough termination or BNC Tee and $50\ \Omega$ termination on one end
  - High-End scopes have both options – check that you use the appropriate
  - High BW scopes only have $50\ \Omega$ input – for high impedance you need active high-impedance adapter

- Bandwidth & Time Domain Specs:
  - For Transducers that work with high-impedance and $50\ \Omega$, specs only guaranteed for one of the two

- There are non-$50\ \Omega$ BNC cables and connectors
  - If length comparable to wavelength, then accuracy is affected

Uncertainty Analysis

For precision measurements

GMW Associates

Measurement Uncertainty Overview

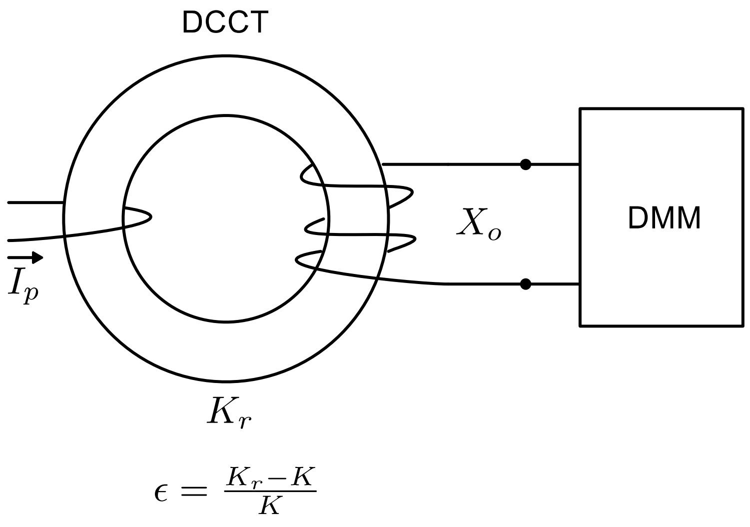
- Need to determine **best estimate** $\bar{X}$ of quantity X referred to as **measurand**
  - The measurement has **uncertainty** $u(X)$
  - **True value** of measurand lies in the interval $\bar{X} \pm u(X)$
  - Note: $\bar{X}$ and $u(X)$ have same units!
- Two types of uncertainty:
  - **Type A:** due to statistical nature of measurement – take 10 samples $u_A(X) = STD(X_1, \ldots, X_{10})$
  - **Type B:** determined by other means – instrument accuracy specs or calibration certificate
- **Combined uncertainty:** $u_C(X) = \sqrt{u_A^2(X) + u_B^2(X)}$
- **Expanded uncertainty:** $u(X) = k \cdot u_C(X)$
  - $k = 2$ for 95% coverage
- **Numerical example:** We are measuring $\sim 2\ \mathrm{A}$ with a DMM
  - From the DMM cal cert for $3\ \mathrm{A}$ range $u(X) = 3 \cdot 10^{-4}\ \mathrm{A} \rightarrow u_B(X) = u(X)/2 = 1.5 \cdot 10^{-4}\ \mathrm{A}$.
  - We take 10 samples with mean $\bar{X} = 1.9995\ \mathrm{A}$ and STD $u_A(X) = 2.9 \cdot 10^{-4}\ \mathrm{A}$
  - The combined uncertainty is $u_C(X) = 3.3 \cdot 10^{-4}\mathrm{A} \rightarrow u(X) = 2 \cdot u_C(X) = 6.6 \cdot 10^{-4}\ \mathrm{A}$
  - **True value** lies in interval $1.9995 \pm 0.00066\ \mathrm{A}$

What about Measurements with Multiple Instruments?

- We need **model of the measurement** $Y = f(X_1, \dots, X_m)$
  - Y is measurand
  - $X_1, \dots, X_m$ are sub-quantities we measure directly as before
- For each sub-quantity we calculate combined uncertainty $u_C(X_1), \dots, u_C(X_m)$
- **Linearize model** around best estimate point $\bar{Y} = f(\overline{X_1}, \dots, \overline{X_m})$
  - Calculate **sensitivity coefficients** $\frac{\partial Y}{\partial X_i}$
  - **Propagate** combined uncertainties $u_C^2(Y) = \sum_i \left(\frac{\partial Y}{\partial X_i}\right)^2 u_C^2(X_i)$
- **Expanded uncertainty** $u(Y) = k \cdot u_C(Y)$
- **True value** of measurand lies in the interval $\bar{Y} \pm u(Y)$

Measuring Current with a Current Transducer and DMM

- Need to **measure current with a DCCT and DMM**
- DCCT has output X_o measured on DMM
- DCCT has **true ratio** $\mathrm{K} = \frac{K_r}{1+\epsilon}$
  - K_r is the rated ratio from manufacturer's spec
  - ϵ is the ratio error found in the DCCT calibration certificate
- From the DMM and transducer K, primary current is $I_p = KX_o$
- The **model of the measurement** is $I_p = \frac{K_r X_o}{1+\epsilon}$
- We take n measurements of X_o – best estimate for DCCT output is $\overline{X_o} = \left\langle X_o^{(i)} \right\rangle$
- **Best estimate** of primary current is $\overline{I_p} = \frac{K_r \overline{X_o}}{1+\epsilon} \approx K_r \overline{X_o}$



GMW Associates

Uncertainty Analysis

- Sources of Uncertainty:
  - **Combined** uncertainty of the **DCCT output** $u_C(X_o)$
  - **Type B** uncertainty of the **DCCT ratio error** $u_B(\epsilon)$
- **Combined** uncertainty of the **DCCT output** $u_C(X_o) = \sqrt{u_A^2(X_o) + u_B^2(X_o)}$
  - **Type A** uncertainty $u_A^2(X_o) = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_o^{(i)} - \overline{X_o}\right)^2$
  - **Type B** uncertainty $u_B(X_o)$ from the accuracy specs or calibration certificate of the DMM
- Primary current **sensitivity coefficients**:
  - $\frac{\partial I_p}{\partial X_o} = \frac{K_r}{1+\epsilon}$
  - $\frac{\partial I_p}{\partial \epsilon} = \frac{K_r X_o}{(1+\epsilon)^2}$
- **Combined** uncertainty of primary current $u_C(I_p) = \sqrt{\left(\frac{\partial I_p}{\partial X_o}\right)^2 u_C^2(X_o) + \left(\frac{\partial I_p}{\partial \epsilon}\right)^2 u_B^2(\epsilon)}$
- **Expanded** uncertainty of primary current $u(I_p) = k\, u_C(I_p)$

Instrument Uncertainties & Setup

- Two options for **Type B** instrument uncertainties $u_B(X_o)$ and $u_B(\epsilon)$:
  - **Manufacturer's specifications** given as $\%\ Reading + \%\ Range$
    - Calculate expanded uncertainty $u(X) = \epsilon_{reading}X_{reading} + \epsilon_{range}X_{range}$
    - Calculate Type B uncertainty $u_B(X) = u(X)/k$
  - **Calibration Certificate**
    - Interpret numbers as expanded uncertainty
    - Calculate Type B uncertainty $u_B(X) = u(X)/k$
- **Note:** When instrument has multiple ranges, **need uncertainty of active range!**
  - It's better to manual set the instrument range than use *Autorange*

Numerical Example

- We are measuring $\sim 3\ \mathrm{kA}$ with a current-output DCCT and a DMM
- DCCT with rate ratio $K_r = 1500$
  - From the calibration certificate $\epsilon = -23\ \mathrm{ppm}$ and $u(\epsilon) = 0.1\% \rightarrow u_B(\epsilon) = u(\epsilon)/2 = 0.5 \cdot 10^{-3}$
- DMM in the $3\ \mathrm{A}$ range (expect $\sim 2\ \mathrm{A}$)
  - From the calibration certificate $u(X_o) = 3 \cdot 10^{-4}\ \mathrm{A} \rightarrow u_B(X_o) = u(X_o)/2 = 1.5 \cdot 10^{-4}\mathrm{A}$
- We take $n = 10$ measurements with the DMM
  - Mean value $\overline{X_o} = 1.9995\ \mathrm{A}$ results in best estimate of primary current $\overline{I_p} = 2999.32\ \mathrm{A}$
  - Sample STD $u_A(X_o) = 2.9 \cdot 10^{-4}\ \mathrm{A}$
- Transducer output combined uncertainty $u_C(X_o) = \sqrt{u_A^2(X_o) + u_B^2(X_o)} = 3.3 \cdot 10^{-4}\ \mathrm{A}$
- Sensitivity coefficients $\frac{\partial I}{\partial X_o} = \frac{K_r}{1+\epsilon} = 1500.03$ and $\frac{\partial I}{\partial \epsilon} = \frac{K_r X_o}{(1+\epsilon)^2} = 2999.39\ \mathrm{A}$
- Propagate $u_C(I_p) = \sqrt{\left(\frac{\partial I_p}{\partial X_o}\right)^2 u_C^2(X_o) + \left(\frac{\partial I_p}{\partial \epsilon}\right)^2 u_B^2(\epsilon)} = 1.58\ \mathrm{A}$ and $u(I_p) = 2 \cdot u_C(I_p) = 3.15\ \mathrm{A}$
- **True value** of primary current lies in interval $2999.32 \pm 3.1\ \mathrm{A}$

Other Measurement Models

DC Offset

- When measuring **DC current**, transducer may exhibit **small zero current offset** X_{off}
  - Flux-gate DCCT **Turn-on history** causes **small zero-offset drifts**.
- Model of Measurement with Offset: $I_p = \frac{K_r(X_o - X_{off})}{1+\epsilon}$
  - Added **combined uncertainty** of zero current **offset measurement**

Using a Burden Resistor

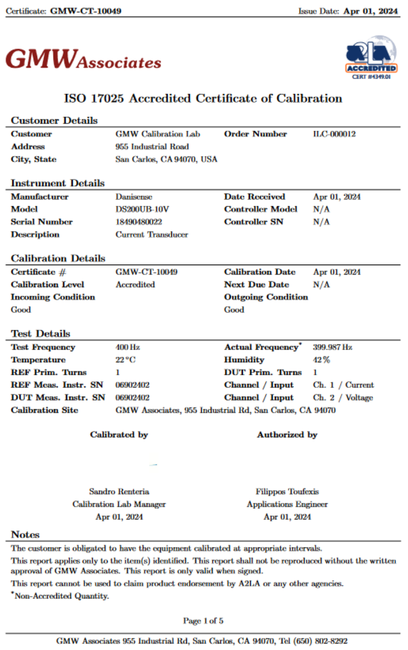
- Often DMM voltage channel has better accuracy than current
  - It may be preferable to measure voltage drop V_o on burden resistor R_B
- Model of Measurement with Burden Resistor: $I_p = \frac{K_r}{1+\epsilon} \frac{V_o}{R_B}$
  - Added **Type B uncertainty** of burden **resistor value**

GMW 17025 Accredited Calibration Certificate

GMW Current Calibration Services:

- **DC** Amplitude up to **11 kA** & **AC Amplitude & Phase** up to **8 kA** and up to **400 Hz**
- **10 days Typical Turnaround**
- Onsite Calibration Services

Cover Page


Certificate: GMW-CT-10049 Issue Date: Apr 01, 2024

GMW Associates

ISO 17025 Accredited Certificate of Calibration

Customer Details

Customer	GMW Calibration Lab	Order Number	ILC-000012
Address	955 Industrial Road		
City, State	San Carlos, CA 94070, USA		

Instrument Details

Manufacturer	Danisense	Date Received	Apr 01, 2024
Model	DS200UB-10V	Controller Model	N/A
Serial Number	18490480022	Controller SN	N/A
Description	Current Transducer		

Calibration Details

Certificate #	GMW-CT-10049	Calibration Date	Apr 01, 2024
Calibration Level	Accredited	Next Due Date	N/A
Incoming Condition		Outgoing Condition	
Good		Good	

Test Details

Test Frequency	400 Hz	Actual Frequency*	399.987 Hz
Temperature	22 °C	Humidity	42 %
REF Prim. Turns	1	DUT Prim. Turns	1
REF Meas. Instr. SN	06902402	Channel / Input	Ch. 1 / Current
DUT Meas. Instr. SN	06902402	Channel / Input	Ch. 2 / Voltage
Calibration Site	GMW Associates, 955 Industrial Rd, San Carlos, CA 94070		

Calibrated by — Sandro Renteria, Calibration Lab Manager, Apr 01, 2024

Authorized by — Filippos Toufexis, Applications Engineer, Apr 01, 2024

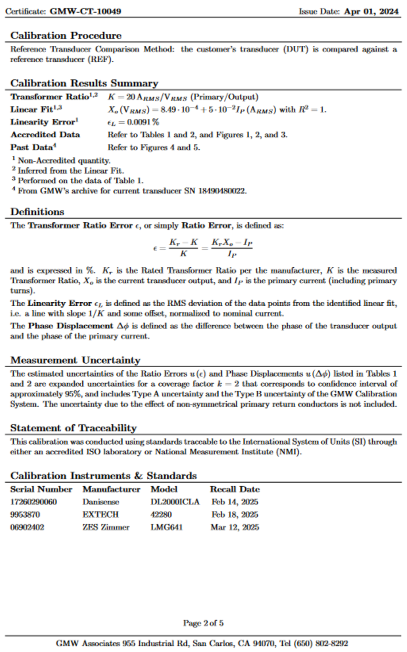
Notes

The customer is obligated to have the equipment calibrated at appropriate intervals.
This report applies only to the item(s) identified. This report shall not be reproduced without the written approval of GMW Associates. This report is only valid when signed.
This report cannot be used to claim product endorsement by A2LA or any other agencies.
*Non-Accredited Quantity.

Page 1 of 5

GMW Associates 955 Industrial Rd, San Carlos, CA 94070, Tel (650) 802-8292

Summary


Certificate: GMW-CT-10049 Issue Date: Apr 01, 2024

Calibration Procedure

Reference Transducer Comparison Method: the customer's transducer (DUT) is compared against a reference transducer (REF).

Calibration Results Summary

Transformer Ratio[1,2]	$K = 20\,A_{RMS}/V_{RMS}$ (Primary/Output)
Linear Fit[1,3]	$X_o(V_{RMS}) = 8.49\cdot10^{-4} + 5\cdot10^{-2} I_P(A_{RMS})$ with $R^2 = 1$
Linearity Error[1]	$\epsilon_L = 0.0091\,\%$
Accredited Data	Refer to Tables 1 and 2, and Figures 1, 2, and 3.
Past Data[4]	Refer to Figures 4 and 5.

[1] Non-Accredited quantity.
[2] Inferred from the Linear Fit.
[3] Performed on the data of Table 1.
[4] From GMW's archive for current transducer SN 18490480022.

Definitions

The **Transformer Ratio Error** ϵ, or simply **Ratio Error**, is defined as:

$$\epsilon = \frac{K_r - K}{K} = \frac{K_r X_o - I_P}{I_P}$$

and is expressed in %. K_r is the Rated Transformer Ratio per the manufacturer, K is the measured Transformer Ratio, X_o is the current transducer output, and I_P is the primary current (including primary turns).

The **Linearity Error** ϵ_L is defined as the RMS deviation of the data points from the identified linear fit, i.e. a line with slope $1/K$ and some offset, normalized to nominal current.

The **Phase Displacement** $\Delta\phi$ is defined as the difference between the phase of the transducer output and the phase of the primary current.

Measurement Uncertainty

The estimated uncertainties of the Ratio Errors $u(\epsilon)$ and Phase Displacements $u(\Delta\phi)$ listed in Tables 1 and 2 are expanded uncertainties for a coverage factor $k = 2$ that corresponds to confidence interval of approximately 95%, and includes Type A uncertainty and the Type B uncertainty of the GMW Calibration System. The uncertainty due to the effect of non-symmetrical primary return conductors is not included.

Statement of Traceability

This calibration was conducted using standards traceable to the International System of Units (SI) through either an accredited ISO laboratory or National Measurement Institute (NMI).

Calibration Instruments & Standards

Serial Number	Manufacturer	Model	Recall Date
17260290060	Danisense	DL2000ICLA	Feb 14, 2025
9953870	EXTECH	42280	Feb 18, 2025
06902402	ZES Zimmer	LMG641	Mar 12, 2025

Page 2 of 5

GMW Associates 955 Industrial Rd, San Carlos, CA 94070, Tel (650) 802-8292

Amplitude Data

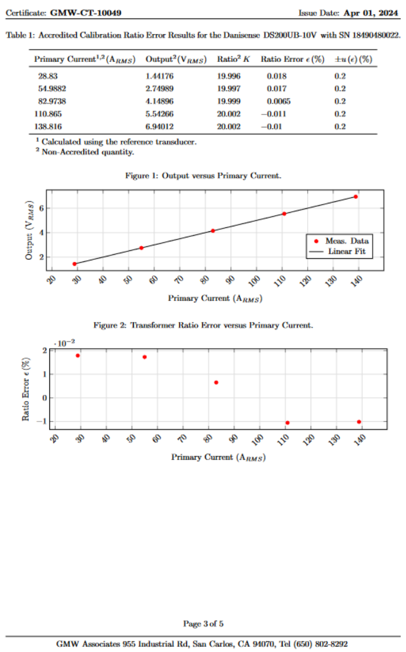

Certificate: GMW-CT-10049 Issue Date: Apr 01, 2024

Table 1: Accredited Calibration Ratio Error Results for the Danisense DS200UB-10V with SN 18490480022.

Primary Current[1,2] (A_{RMS})	Output[2] (V_{RMS})	Ratio[2] K	Ratio Error ϵ (%)	$\pm u(\epsilon)$ (%)
28.83	1.44176	19.996	0.018	0.2
54.9882	2.74989	19.997	0.017	0.2
82.9738	4.14896	19.999	0.0065	0.2
110.865	5.54266	20.002	−0.011	0.2
138.816	6.94012	20.002	−0.01	0.2

[1] Calculated using the reference transducer.
[2] Non-Accredited quantity.

Figure 1: Output versus Primary Current.

Figure 2: Transformer Ratio Error versus Primary Current.

Page 3 of 5

GMW Associates 955 Industrial Rd, San Carlos, CA 94070, Tel (650) 802-8292

Phase Data

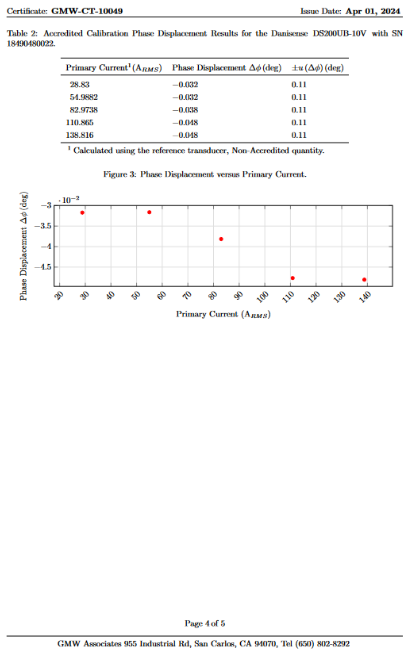

Certificate: GMW-CT-10049 Issue Date: Apr 01, 2024

Table 2: Accredited Calibration Phase Displacement Results for the Danisense DS200UB-10V with SN 18490480022.

Primary Current[1] (A_{RMS})	Phase Displacement $\Delta\phi$ (deg)	$\pm u(\Delta\phi)$ (deg)
28.83	−0.032	0.11
54.9882	−0.032	0.11
82.9738	−0.038	0.11
110.865	−0.048	0.11
138.816	−0.048	0.11

[1] Calculated using the reference transducer, Non-Accredited quantity.

Figure 3: Phase Displacement versus Primary Current.

Page 4 of 5

GMW Associates 955 Industrial Rd, San Carlos, CA 94070, Tel (650) 802-8292

Historic Data

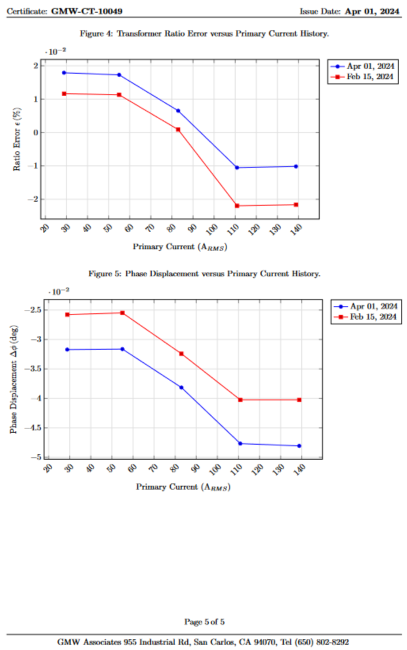

Certificate: GMW-CT-10049 Issue Date: Apr 01, 2024

Figure 4: Transformer Ratio Error versus Primary Current History.

Figure 5: Phase Displacement versus Primary Current History.

Page 5 of 5

GMW Associates 955 Industrial Rd, San Carlos, CA 94070, Tel (650) 802-8292

Further Reading

GMW Associates

Further Reading

General Current Measurement:

- Bastos, M. C. (2016). High precision current measurement for power converters. *arXiv preprint arXiv:1607.01584*. https://arxiv.org/pdf/1607.01584
- Webber, R. C. (1995, May). Charged particle beam current monitoring tutorial. In *AIP Conference Proceedings CONF− 9410219* (Vol. 333, No. 1, pp. 3-23). American Institute of Physics. https://inis.iaea.org/collection/NCLCollectionStore/_Public/26/033/26033463.pdf
- Webber, R. C. (2000, November). Tutorial on beam current monitoring. In *AIP Conference Proceedings* (Vol. 546, No. 1, pp. 83-104). American Institute of Physics. https://lss.fnal.gov/archive/2000/conf/Conf-00-119.pdf

Active (Hereward) Transformer:

- Sharp, J. B. (1962). *The induction type beam monitor for the PS: Hereward transformer* (No. MPS-Int-CO-62-15). https://cds.cern.ch/record/1068123/files/cer-002723994.pdf

Rogowski Coils:

- Ray, W. F., & Hewson, C. R. (2000, October). High performance Rogowski current transducers. In *Conference Record of the 2000 IEEE Industry Applications Conference. Thirty-Fifth IAS Annual Meeting and World Conference on Industrial Applications of Electrical Energy (Cat. No. 00CH37129)* (Vol. 5, pp. 3083-3090). IEEE. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=83bfb38f5370c6744240b21d12f7f1b0fddb0d33

Fluxgate DCCTs:

- Unser, K. B. (1992, April). The parametric current transformer, a beam current monitor developed for LEP. In *AIP Conference Proceedings* (Vol. 252, No. 1, pp. 266-275). American Institute of Physics. https://cds.cern.ch/record/231598/files/CM-P00061084.pdf
- Musmann, G. (Ed.). (2010). *Fluxgate magnetometers for space research*. BoD–Books on Demand.
- Geyger, W. A. (1954). *Magnetic-amplifier circuits: basic principles, characteristics, and applications*. McGraw-Hill.

Further Reading

Transducer Termination / Transmission Line Theory:

- Toufexis, F. (2024). *GWM Application Note: Transducer Termination Impedance – Low and High Frequency Analysis*. https://gmw.com/wp-content/uploads/2024/09/ftouf_CTTerminationTransLine_AppNote_240925.pdf
- Pozar, D. M. (2021). *Microwave engineering: theory and techniques*. John wiley & sons.
- Collin, R. E. (2007). *Foundations for microwave engineering*. John Wiley & Sons.

Uncertainty Analysis:

- Toufexis, F. (2024). *GWM Application Note: Estimating the Current Measurement Uncertainty when utilizing Current Transducers*. https://gmw.com/wp-content/uploads/2023/04/ftouf_CurrentUncertainty_AppNote_240925.pdf
- Taylor, J. R. (1997). An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements.
- Crowder, S., Delker, C., Forrest, E., & Martin, N. (2020). *Introduction to statistics in metrology*. Cham, Switzerland: Springer.
- https://www.bipm.org/en/publications/guides/

GMW Associates

Thank You!

Filippos Toufexis, PhD

filippos@gmw.com

www.gmw.com

650-282-2335

GMW Associates