Application Note: A Guide to Error Analysis in Danisense DCCT Systems and Considerations for Output Selection

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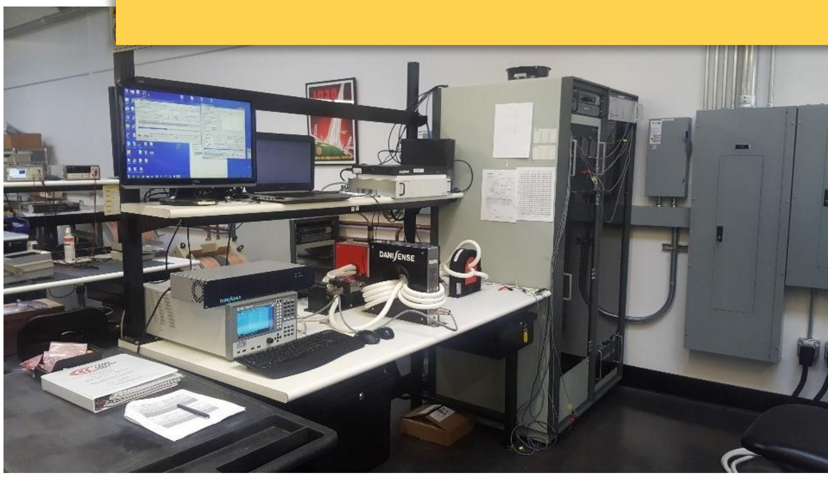

Overview

Danisense DC Current Transducers (DCCTs) offer excellent accuracy and high dynamic range. Several customers use these transducers for critical current measurements that often require error budgets. Danisense datasheets provide several error specifications that can feel complicated for users with no intimate Fluxgate DCCT knowledge. This Application Note explains the different specifications and how to establish an error budget.

The accuracy specifications can vary significantly between current-output and voltage-output DCCTs, and there is also the option of using current-output DCCTs with a Voltage Output Module (VOM) on the DSSIU side. However, selecting between these three options involves not only the accuracy specifications, but also the measurement setup and test area. This note will further dive into these considerations.

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Equipment

- Danisense Transducer
- Measurement Instrument such as a Power Analyzer or DMM

Applications

- High-Accuracy Current Measurements with Danisense DCCTs

Introduction

Current transducers are, to a first approximation, characterized by a transformer or turns ratio N or transfer ratio k where the output current or voltage X_o relates to the primary current I_P according to

$$X_o = \frac{I_P}{k} = \frac{I_P}{N}.$$

(1)

Note we will be using k and N interchangeably. Danisense DCCTs have ppm-level accuracy and the model of equation (1) is a very good approximation when the transducer is operated within its specifications, i.e. primary current range, bandwidth, and temperature.

However, small imperfections in the cores and electronics cause the transfer ratio k to have variations with frequency f and primary current I_p, and in addition there is a very small offset X_{OE} under zero primary current. Because of these imperfections, the following is a more realistic model of the transducer:

$$X_o = \frac{I_P}{k(f, I_P)} + X_{OE}.$$

(2)

The model becomes even more complex when using a current-output transducer with a Voltage Output Module (VOM) in a DSSIU that has its own gain and offset. In addition to the imperfections of the transducer, and potentially of a VOM, the environmental conditions, i.e. the operating temperature T_{env}, magnetic field B_{env}, and power supply voltage V_s, also affect the offset X_{OE} and the transfer ratio (for voltage-output). The following is an even more realistic model of the transducer:

$$X_o = \frac{I_P}{k(f, I_P, T_{env})} + X_{OE}(T_{env}, B_{env}, V_s).$$

(3)

The model of equation (3) is impractical to characterize and use in measurements as it will vary between individual transducers of the same model. Danisense publishes detailed accuracy specifications so that users can compose an **error budget**, i.e. how much they can expect an actual transducer to deviate from the ideal model of equation (1). This way users can benefit from the simplicity of the ideal model of equation (1) because of the accuracy of Danisense sensors, while still quantifying the errors they can expect in a measurement.

The accuracy specifications can vary significantly between current-output and voltage-output transducers. The measurement instrument also needs to be factored in before selecting between a current-output or voltage-output model. Yet another option is using a current-output transducer with a Voltage Output Module (VOM) on the DSSIU side that has similar specifications to a voltage-output transducer. The measurement setup and test area, and the long-term needs of the customer need to get evaluated in selected the appropriate solution.

This Application Note is organized as follows: We begin by explaining the principle of operation of Closed-Loop Fluxgate DC Current Transducers (DCCTs). This discussion sets the stage for subsequently discussing the different sources of errors and the notation used in the accuracy specifications. We proceed to calculate error budgets both analytically and with specific numerical examples for current-output, voltage-output, and current-output transducers with VOMs. As will become clear from this analysis, the offset is a large preventable source of error, and we further discuss how to systematically remove it. We will then discuss some considerations for the instrument used to measure the output of a DCCT, as well as the effect of that instrument on noise pickup from cables. Finally, we present a summary of the main considerations in selecting the right transducer output for a specific application. While readers could jump directly at the examples sections and simply follow the calculations, it is recommended that they read the entire Note to get a deeper understanding of these sensors.

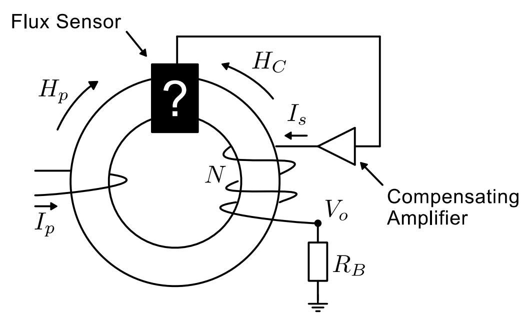

Figure 1: Closed-loop or zero-flux current transducer concept.

Closed-Loop Fluxgate Principle of Operation

Danisense DCCTs are closed-loop fluxgate current transducers, and this section describes their operation starting from basic principles. The closed-loop sensor, also referred to as zero-flux sensor, is a current transducer with a magnetic core and some "flux sensor" to measure the flux in the core because of a primary current I_p going through the aperture, as shown in Figure 1. A feedback loop using a compensating amplifier drives an opposing current I_s into a secondary winding of N turns, maintaining zero-flux in the core. The primary current I_p is measured indirectly by measuring the secondary current $I_s = I_p/N$. The secondary current is measured either directly or through a burden resistor R_B. Often a DMM or Power Analyzer is used for high accuracy.

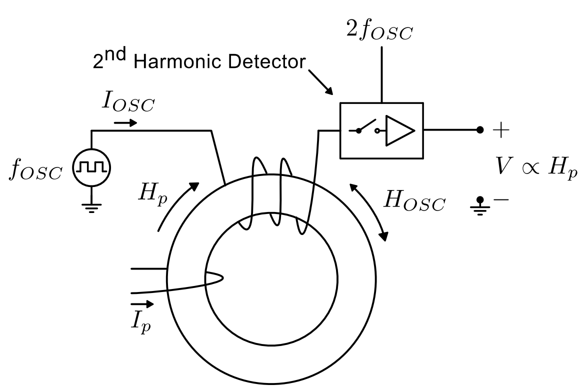

Figure 2: Flux-gate detector concept.

One option for this flux sensor is cutting a small gap in the core and inserting a Hall-effect sensor into the gap. Current transducers based on this approach are low-cost but have mediocre accuracy, temperature changes cause mechanical changes, and they are sensitive to external fields due to the gap. Another option is to use another coil that is referred to as the Hereward Transformer [1]; however, this is AC-only. Yet another option is the fluxgate detector shown schematically in Figure 2.

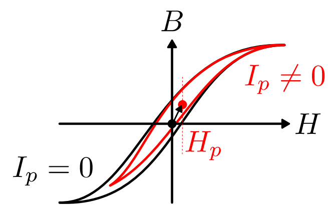

Figure 3: The effect of a primary current in the B-H loop.

The fluxgate detector is an extremely sensitive detector of DC and low-frequency magnetic flux and is based on modulating the magnetic permeability of the core. If we have a winding on a core, then the Electromotive Force, or induced voltage, is given by

$$\mathcal{EMF} = -NA\frac{d}{dt}[\mu(t)H(t)]. \tag{4}$$

It is evident that for a DC magnetic field $\frac{d}{dt}H(t) = 0$. The only way to have a non-zero induced voltage is $\frac{d}{dt}\mu(t) \neq 0$, i.e. to detect a DC magnetic field through a core winding, we need to modulate the permeability of the core. Therefore, the first component of a fluxgate detector is a circuit to drive the magnetic core in and out of deep saturation, at some oscillator frequency f_{osc}, by driving an alternating current through a separate excitation winding. As will be subsequently explained, this results in another signal at the second harmonic of f_{osc} that is proportional to the primary current. This signal is detected by a 2nd Harmonic Detector, as shown in Figure 2.

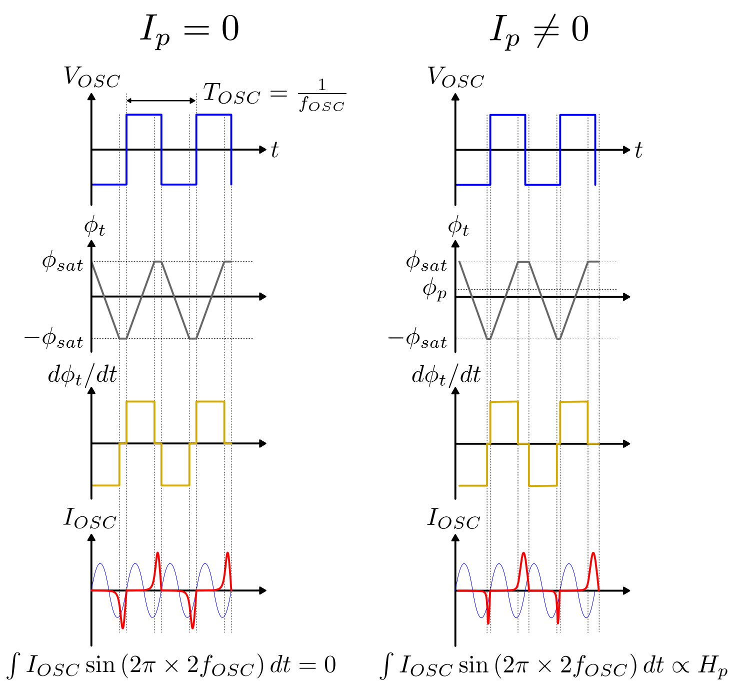

Figure 4: Fluxgate excitation curves. The blue curve on bottom row is a sine at frequency $2f_{osc}$ to show the overlap with the current I_{osc} (red curve).

When the core is far from saturation it has its nominal material permeability; however, when it is saturated, it has a very small permeability close to that of free space. Under zero primary current, the excitation circuitry is cycling the core through its major B-H loop symmetrically between positive and negative saturation. When finite primary current is present, and therefore finite magnetic field H_p, the B-H loop is biased to one side and is no longer symmetric, as illustrated in Figure 3.

We can get some intuitive understanding of the process by looking at the simplified curves of the various magnetic parameters shown in Figure 4, assuming the winding is driven by a square voltage pulse. When the voltage pulse changes polarity, the magnetic flux and current rise linearly until saturation. When the saturation flux is reached, the magnetic permeability collapses and therefore the impedance of the coil drops, and there is a current spike. The bottom figures of Figure 4 show the current through the coil and a sine at the second harmonic of the drive frequency. When there is no primary current, these parameters vary symmetrically between positive and negative saturation. Therefore, the integral of the current waveform and the second harmonic sine is zero, i.e. the current waveform has no second harmonic component. However, when there is finite primary current, the B-H curve is biased to one polarity, and the core spends more time in saturation on that polarity. As a result, more current flows on one polarity and the integral of the current waveform and the second harmonic sine is not zero. There is a finite second harmonic component, that, as will be shown later, is proportional to the field H_p and therefore the primary current I_p.

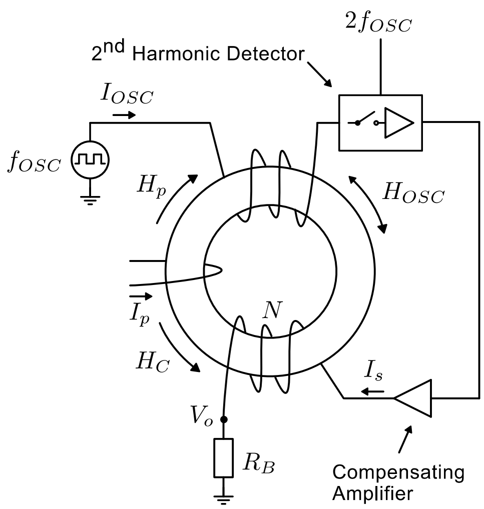

Figure 5: Closed-loop fluxgate current transducer concept.

Combining the concept of the zero-flux sensor with the flux-gate detector yields the closed-loop flux-gate current transducer shown in Figure 5. The magnetic core is driven in and out of saturation by an excitation circuit and an excitation winding. The primary current going through the aperture is detected by a second harmonic detector. The output of the second harmonic detector drives a compensating amplifier and a current through a compensation winding to zero the flux of the core. The primary current I_p is measured indirectly by measuring the compensation current $I_s = I_p/N$. Because the **flux-gate detector is extremely sensitive** and the whole **system is linearized** around the zero-flux point of the core due to the feedback loop, the **resulting sensor is extremely accurate and linear**. Unlike the gaped closed-loop transducers based on Hall-effect sensors mentioned above, there is **reduced magnetic coupling** from adjacent currents because all the cores are closed, e.g. as with three sensors measuring the phase currents of a 3-phase power system.

The schematic of Figure 5 is impractical because of the excitation signal breakthrough from the excitation coil to the compensation coil and limited bandwidth. In general, single core sensors are additionally sensitive to temperature drifts. Figure 6 shows the practical implementation of a closed-loop flux-gate current transducer. Such sensors were originally invented at CERN to measure particle beam current [2]. Two cores with matched properties are used to cancel temperature drifts and the excitation breakthrough. The two cores are modulated in opposition while the 2nd harmonic demodulator takes the difference of the signals from the two cores to drive the compensation current I_s. There are several excitation and demodulation circuit options that result in the same system functionality. In this Application Note we do not focus on a specific implementation. Instead, we make some assumptions as needed to help with the explanation of errors originating from the physics of the sensor.

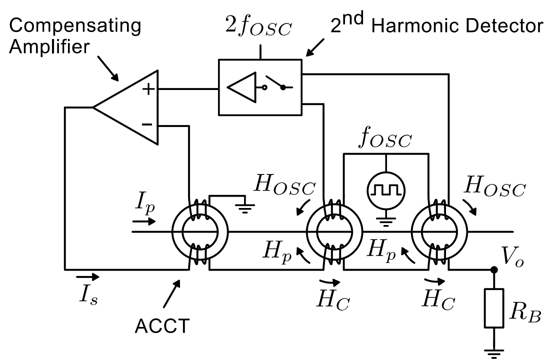

Figure 6: Practical flux-gate current transducer implementation.

The fluxgate part of the sensor has bandwidth on the order of 100 Hz to 1 kHz. A third core is used as an AC current transformer (ACCT) to detect higher frequency signals, while being part of the feedback loop. The compensating amplifier sums the low-frequency signal from the fluxgate and the high-frequency signal from the ACCT and drives the compensation current through all three cores. As a result, the ACCT is not affected by a large DC primary current as its flux is canceled by the fluxgate and feedback loop. We can see four regions of operation in the frequency domain:

1. DC and very low frequency: the fluxgate and amplifier drive I_s.
2. Low frequency: the amplifier sums the fluxgate and ACCT signals to drive I_s.
3. Medium frequency: the amplifier only uses the ACCT signal to drive I_s.
4. High frequency: the amplifier has reached its unity gain and the ACCT with the burden resistor behaves as a passive current transformer.

We will now look closely at the fields of the two cores of the fluxgate to understand several error sources using the schematic of Figure 7. Each core sees the same azimuthal magnetic field H_p due to the primary current I_p, while the excitation field H_{OSC} is opposite between them. A perfectly uniform external field would not induce a net azimuthal field in a perfectly cylindrical core due to symmetry; however, if such an external field had a gradient then a net azimuthal field could develop. In either case and depending on the orientation, a transverse field component can get developed. For the purposes of this analysis, we will model an external field acting on the sensor, such as the earth's magnetic field, as an additional field component H_{ext}, transverse to the plane of the toroid. Now the total magnetic field in the core is:

$$H = \sqrt{\left(H_p \pm H_{OSC}\right)^2 + H_{ext}^2},$$

(5)

where the $\pm$ refers to the two cores excited in opposition. We can simplify (5) using the binomial approximation $\sqrt{1+x} \approx 1 + x/2$ for $|x| \ll 1$. Assuming H_{ext} is very small compared to $H_p \pm H_{OSC}$, equation (5) becomes

$$H = \left(H_p \pm H_{OSC}\right)\sqrt{1 + \left(\frac{H_{ext}}{H_p \pm H_{OSC}}\right)^2} \approx \left(H_p \pm H_{OSC}\right)\left[1 + \frac{1}{2}\left(\frac{H_{ext}}{H_p \pm H_{OSC}}\right)^2\right].$$

(6)

Further approximating $x^2 \approx x$ for $|x| \ll 1$, equation (6) becomes

$$H \approx H_p \pm H_{OSC} + \frac{1}{2} H_{ext}.$$

(7)

This is the total field amplitude in the core due to both azimuthal and longitudinal components. We will model the B-H loop of the cores according to

$$B = f(H) = \alpha H - \beta H^3,$$

(8)

which is a simplified model to help with the analysis. Coefficients α and β depend on the specific core material, and two cores, despite being of the same material, can have slightly different coefficients because of slightly different material properties or excitation history.

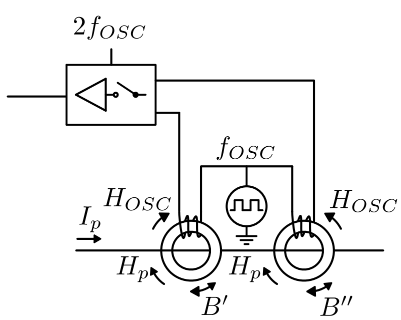

Figure 7: Field analysis in the 2-core flux-gate current transducer.

We can now model the magnetic flux density in the two cores as

$$B' = f\left(H_p + H_{OSC} + \frac{1}{2}H_{ext}\right) = \alpha'\left(H_p + H_{OSC} + \frac{1}{2}H_{ext}\right) - \beta'\left(H_p + H_{OSC} + \frac{1}{2}H_{ext}\right)^3$$

$$B'' = f\left(H_p - H_{OSC} + \frac{1}{2}H_{ext}\right) = \alpha''\left(H_p - H_{OSC} + \frac{1}{2}H_{ext}\right) - \beta''\left(H_p - H_{OSC} + \frac{1}{2}H_{ext}\right)^3$$

(9)

and assume that the excitation field varies as

$$H_{OSC} = H_M \sin \omega t + H_{off}.$$

(10)

In equation (10), H_M is the amplitude of the modulation that depends on the power supply voltage, and H_{off} is an offset from perfectly symmetrical excitation due to a power supply imbalance or other errors in the electronic circuit.

If we added a pickup coil of N turns to both cores in series, then the total induced voltage would be

$$EMF = -NA\frac{d}{dt}(B' + B''),$$

(11)

where A is the cross-section of the two cores. We are interested in calculating the amplitude of the second harmonic signal. For simplicity we subsequently outline the steps of the analytical derivations without showing in-between results; an analytical software package like Mathematica can be used to replicate these calculations.

We start by substituting equation (10) into equations (9). We then plug equations (9) into equation (11), further replacing

$$\alpha'' + \alpha' = 2\alpha,$$

$$\alpha'' - \alpha' = \epsilon_\alpha,$$

$$\beta'' + \beta' = 2\beta,$$

$$\beta'' - \beta' = \epsilon_\beta,$$

(12)

where the terms ϵ_α and ϵ_β correspond to mismatches in magnetic properties between the two cores. After simplifying the trigonometric power functions into harmonics, we take the second harmonic term $\sin(2\omega t)$:

$$EMF|_{2\omega} = 3NA\omega H_m^2\left[\left(\beta H_{ext} - \epsilon_\beta H_{off}\right) + (2\beta)H_p\right]\sin(2\omega t).$$

(13)

Equation (13) tells up a couple of important things: First, if $\beta = 0$ then $EMF|_{2\omega} = 0$ and therefore there would be no second harmonic signal without the nonlinearity of the core. Second, if everything was perfect, i.e. $H_{ext} = 0$, $\epsilon_\beta = 0$, and $H_{off} = 0$, then the second harmonic signal and therefore the output signal of the second harmonic detector output would be directly proportional to H_p. The feedback loop would try to zero the second harmonic detector output by generating a compensation current that is also proportional to H_p, and therefore proportional to the primary current I_p. However, because of the errors previously mentioned, there would be an additional **offset** at the output of the second harmonic detector proportional to $\beta H_{ext} - \epsilon_\beta H_{off}$ that **does not change with current**. The feedback loop would also try to correct the offset, causing an **offset compensation current**. This is potentially the most critical piece information to understand for making accurate measurements with closed-loop fluxgate transducers as we will discuss in the rest of this Application Note.

Finally, we need to address the operation of voltage-output closed-loop fluxgate sensors. So far, we have described the operation of current-output closed-loop fluxgate sensors, where the output or secondary or compensation current I_s relates to the primary current I_p precisely (except for the error sources described subsequently) via the number of turns N of the compensation winding according to

$$\frac{I_p}{I_s} = N, \tag{14}$$

and is expressed in $\mathrm{A/A}$ or $\mathrm{Amperes\ of\ Primary/Amperes\ of\ Secondary}$. The current-output sensor behaves effectively like a transformer, except that it also responds to very low frequency and purely DC current, and hence the term Direct-Current Current Transformer (DCCT). The user of these sensors measures directly the secondary current I_s via the current input of a DMM or Power analyzer and multiplies by the number of turns N according to equation (14) to measure indirectly the primary current I_p. If instead of directly measuring current, the user desires to perform a voltage measurement, then a voltage output transducer must be used. We will discuss the reasons for doing this later in this Application Note. A voltage output transducer is essentially a current output transducer with an added Voltage Output Module (VOM) converting the output or secondary current I_s into a voltage output V_o that can be measured via the voltage input of a DMM, Power Analyzer, or an Oscilloscope. The VOM is characterized by a transfer ratio

$$k^{VOM} = \frac{I_s}{V_o}, \tag{15}$$

following the conversation of the turns ratio of equation (14). The system transfer ratio of the current output transducer and VOM is given by

$$k = \frac{I_p}{V_o} = \frac{I_p}{I_s}\frac{I_s}{V_o} = N \cdot k^{VOM}, \tag{16}$$

and is expressed in units of A/V or $\mathrm{Amperes/Volts}$. The VOM can be a simple burden resistor or a more complicated signal conditioning circuit with an amplifier. In the case of a simple burden resistor R_B, then the VOM transfer ratio is $k^{VOM} = 1/R_B$ and the system transfer ratio is $k = N/R_B$. The presence of a VOM adds additional components with their own sources of errors that are additionally affected by the environment, such as the temperature or the power supply, as we will discuss subsequently. Note that in the following for simplicity we may use the turns ratio N and transfer ratio k interchangeably; both denote the ratio of the primary current I_p and the output signal X_o, whether that output signal is a secondary current I_s or an output voltage V_o.

Sources of Error & Notation

Now that we understand the physics of closed-loop fluxgate transducers, we can discuss the different errors. Danisense specification sheets contain several numbers as error specifications that can overwhelm users. To explain these errors and provide a concise methodology for calculating an error budget, it is imperative to establish a notation convention that is as close as possible to the notation used on the Danisense specifications.

First let's define the problem we are trying to solve, i.e. the error budget: We want to measure a true primary current I_p using a specific current transducer. Because of several potential errors coming from the transducer, as well as the effect of the environment, the measurement yields a slightly different current I_p'. The error budget is the interval $I_p \pm \Delta I_p$ that we can expect the measured current I_p' to lie in. The narrower this interval, the higher the confidence in our measurement. We therefore need to calculate the term ΔI_p using the specification sheet at the current I_p we are interested in measuring. We can look at the term ΔI_p as an absolute number, or as a ratio $\Delta I_p / I_p$. We can also divide it by the turns ratio N or transfer ratio k of the transducer and instead look at the output current or voltage interval $X_o \mp \Delta X_o = \left(I_p \pm \Delta I_p\right)/k$ that we expect to measure with a measurement instrument. It is up to the user to choose which form satisfies their objective.

One thing to note is we are referring to calculations at one frequency, including DC, and that I_p, ΔI_p, X_o, ΔX_o refer to Root Mean Square (RMS) signal quantities. Also note that the amplitude and RMS of a DC signal are the same. There will be no distinction between DC and AC in the following calculations, and there will be no RMS subscript, except for when the transducer nominal current needs to be evaluated; for that we will have to use either the DC or AC nominal current. Another thing to note is that if we have a complex waveform, such as a triangular inductor ripple current waveform, then that waveform is decomposed via the Fourier Transform into a DC component and several pure sinusoidal signals at different frequencies, each with its own amplitude and phase. This situation requires we calculate $\Delta I_p = \Delta I_p(f)$ as a function of the frequency f, i.e. we need the error budget spectrum; this will be out of the scope of this Application Note. In the simple case where we are measuring a purely sinusoidal current on top of a DC current we can do the analysis separately for the AC and DC components.

It is worth distinguishing between the error budget and the measurement uncertainty. Both the error budget and the measurement uncertainty essentially allude to the accuracy of measurement; however, the error budget is a prediction of a future measurement with imaginary devices while the uncertainty is a post-mortem of a measurement that already happened with specific devices. In the error budget, we focus on the transducer abstractly and look at the different effects that could affect a measurement and try to **predict** how good that measurement **will be** using the specification sheets. In the measurement uncertainty, we have a physical transducer in an actual environment and affected by the environment, with very specific, yet unknown, errors that are partially characterized by a calibration. We are using the word "partially", because the calibration got performed in a different environment at a different time, and while conditions are well-controlled in calibration houses, they will not match the operating conditions. We are measuring the transducer output with a physical instrument, which may or not be in the same environment as the transducer, that is also affected by the environment and having its own errors that have also been partially characterized by another calibration. Some transducers have separate head and electronics that may be installed in different environments. We take several data points that in principle are obtained under the same conditions and excitation, but they are all ever so slightly different. We then piece together specifications and/or calibration certificate data of both the transducer and the measurement instrument, as well as the statistics of the measurement, to **quantify** how good the measurement **was**.

In the rest of this section, we will introduce the general notation and discuss the different sources of error individually, and for each, the effects of the environment. In the next two sections we will discuss how to properly add the different errors analytically for the different transducer output options, and present numerical examples.

Notation

We have a specific Danisense current transducer model with turns ratio or transfer ratio k and Nominal Primary Current I_{PN} ($I_{PN\,DC}$ for DC and $I_{PN\,AC}$ for AC in the datasheet). Unless otherwise noted for a specific case, this transducer produces a current or voltage output X_o for primary current I_p.

Except for the total accuracy error, several relative errors are specified in parts-per-million ppm of either the reading or the full scale. Except for the Offset, the relative error due to effect X is denoted by ϵ_X. The relative error ϵ_X may additionally be affected by several environmental parameters Y_i. The effect of environmental parameters Y_i on relative error ϵ_X is denoted by a sensitivity coefficient $\partial\epsilon_X/\partial Y_i$ listed in the datasheet in units of $\mathrm{ppm}/Y_i\,\mathrm{units}$. Therefore, if the environmental parameter Y_i deviates ΔY_i from nominal, e.g. temperature deviation from 23^oC, then the relative error ϵ_X increases by

$$\frac{\partial\epsilon_X}{\partial Y_i}\,\Delta Y_i, \tag{17}$$

in units of ppm. The total relative error $\epsilon_{X,t}$ due to effect X and the environment is calculated as

$$\epsilon_{X,t} = \epsilon_X + \sum_i \frac{\partial\epsilon_X}{\partial Y_i}\,\Delta Y_i. \tag{18}$$

If the relative error due to effect X is not affected by the environment, either because there is no relevant specification or because we operate at nominal conditions, then $\epsilon_{X,t} = \epsilon_X$.

We can convert this total relative error to an absolute error referenced to the primary current by multiplying with 10^{-6} and with the primary current reading I_p or full scale I_{PN}, depending on the specification. This results in a number expressed in $\mathrm{Amperes}$. The term 10^{-6} is to convert parts-per-million into a unitless ratio. We will denote the total absolute error due to effect X referenced to the primary current as $\epsilon_{X,t}\big|^p$, and referred to the secondary or output as $\epsilon_{X,t}\big|^o$. The absolute error referenced to the secondary is expressed in units of $\mathrm{Amperes}$ for current-output transducers and Volts for voltage-output transducers. It follows from equation (1) that the two total absolute errors relate according to

$$\epsilon_{X,t}\big|^o = \frac{\epsilon_{X,t}\big|^p}{k}. \tag{19}$$

The above might seem confusing; however, this framework of capturing errors allows us to shift our perspective of an error; we can see an error as a relative quantity, as a change in primary current, or a change in measured output. We should therefore develop some intuition about how to interpret the relative error $\epsilon_{X,t}$ and the absolute errors $\epsilon_{X,t}\big|^{p}$ and $\epsilon_{X,t}\big|^{o}$ referenced to the primary and secondary respectively. Let's imagine we have a transducer that has slightly higher transfer ratio k' than the specification k, i.e. $k' > k$. That means that if we measure an output signal X_o, the true primary current

$$I_p' = k' X_o \tag{20}$$

will be higher than the measured current using equation (1). We can model this using the total relative error $\epsilon_{X,t}$ as

$$k' = k\left(1 + \epsilon_{X,t}\right) \rightarrow k = \frac{k'}{1+\epsilon_{X,t}}, \tag{21}$$

where we have assumed $\epsilon_{X,t}$ is a unitless ratio. Plugging (21) into (1) yields

$$I_p\left(1 + \epsilon_{X,t}\right) = k' \cdot X_o \rightarrow I_p + \epsilon_{X,t}\big|^{p} = k' \cdot X_o \tag{22}$$

Equation (22) shows us $\Delta I_p = I_p' - i_p = X_o(k' - k) = \epsilon_{X,t}\big|^{p}$, i.e. the true current is larger than what we would expect from the specifications by $\epsilon_{X,t}\big|^{p}$ $\mathrm{Amperes}$. We can also re-arrange (22) using equation (19) as follows:

$$I_p = k\left(X_o - \frac{\epsilon_{X,t}\big|^{p}}{k}\right) = k\left(X_o - \epsilon_{X,t}\big|^{o}\right) \tag{23}$$

Equation (23) shows us $\Delta X_o = -\epsilon_{X,t}\big|^{o}$, i.e. the measured output given a true primary I_p will be lower than what we would expect from the specifications by $\epsilon_{X,t}\big|^{o}$ $\mathrm{Amperes}$ or Volts.

Note that in our measurements we are using the linear model of (1) that has no offset terms. In the Danisense specifications, all **errors** are evaluated at one **specific current point** I_p and therefore are modeled as an **effective change in transfer ratio**, whether they scale with current or not. As a result, any error can be expressed either as a relative error in the transfer ratio $\epsilon_{X,t}$, or as an absolute error in the measurement of primary current $\epsilon_{X,t}\big|^{p}$, or as an absolute error in the expected output signal $\epsilon_{X,t}\big|^{o}$.

To summarize the above, to get the error due to effect X, we get from the datasheet the respective relative error ϵ_X, and add any environmental effects according to (18). This gives us the total relative error $\epsilon_{X,t}$ in units of ppm. We then multiply by the primary current reading I_p or full scale I_{PN} to get the absolute error of primary current in $\mathrm{Amperes}$ $\epsilon_{X,t}\big|^{p}$. We then **sum all the relevant errors** to get the maximum ΔI_p. In the following, we discuss specific error sources.

Linearity Error

The Linearity Error refers to the deviation of the transducer's output from a perfectly linear behavior with increasing primary current. As previously mentioned, closed-loop fluxgate transducers are extremely linear because the fluxgate sensor is linearized around the zero-flux point of the core by the feedback loop. However, they are not perfectly linear. Non-linearities of the different electronic components and interactions between them will give rise to some small nonlinearity at the ppm level. The Linearity error is a relative error denoted by ϵ_L and is not affected by the environment, i.e. $\epsilon_{L,t} = \epsilon_L$. To convert this total relative Ratio error into an absolute error referenced to the primary of the current transducer we multiply with the primary current

$$\epsilon_{L,t}\Big|^p = \epsilon_{L,t} \cdot 10^{-6} \cdot I_p.$$

(24)

To better understand the Linearity Error, let's imagine we took several measurements of the output X_o^i of a transducer at different primary currents I_p^i. We fit a linear model of the form

$$I_p(X_o) = k' \cdot X_o - I_{off},$$

(25)

where k' is the transfer ratio (but not necessarily the exact specification), and I_{off} is the offset current from the fit. The Linearity Error is the expected difference between the actual reading I_p^i and the model fit $I_p(X_o^i)$ at that data point. If we express it as an absolute error referred to the primary we have:

$$\epsilon_L|^p = I_p^i - I_p(X_o^i).$$

(26)

This is illustrated in Figure 8.

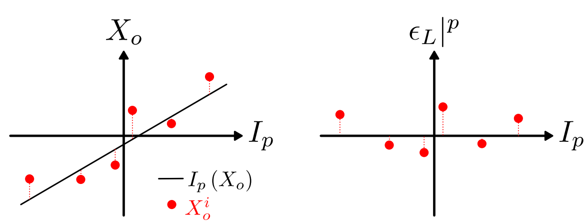

Figure 8: Graphical illustration of the Linearity Error.

Offset

The Offset refers to a DC output signal present when the transducer is powered on and there is zero primary current. When there is finite primary current, this offset is superimposed to the signal generated by the primary current thus corrupting the measurement. It is often the largest error source, and because it does not scale with current, it can dominate the error budget at low currents. However, it is also the only error source we can systematically eliminate given certain conditions, as we will discuss in a subsequent section.

As we saw from equation (13) when discussing the detailed operation of closed-loop fluxgate sensors, slight imperfections of the magnetic cores and the electronics, as well as the environment, will cause the second harmonic detector to generate an offset signal that does not scale with the primary current. This offset signal from the second harmonic detector will propagate through the feedback loop into the transducer's output. The offset error is a relative error denoted by I_{OE}.

Equation (13) further alludes to the different environmental factors affecting the offset and are included in the Danisense datasheets. The coefficient H_{ext} in equation (13) tells us that the offset is affected by an external magnetic field. After all, a closed-loop fluxgate transducer is effectively a very sensitive fluxgate magnetometer [3]. The effect of the earth's magnetic field is included in the I_{OE} specification; however, any additional magnetic field ΔB will result in an offset error contribution of

$$\frac{\partial I_{OE}}{\partial B}\,\Delta B, \tag{27}$$

where $\partial I_{OE}/\partial B$ is the sensitivity coefficient to the magnetic field found in the datasheet and is typically expressed in $\mathrm{ppm/mT}$. This is a simplification to capture the worst case; in reality the effect of ΔB also depends on the orientation with respect to the transducer. In addition, an imbalance in the excitation signal due to power supply variations ΔV, i.e. excitation offset H_{off} in equation (13), coupled with a mismatch in magnetic properties between the two cores ϵ_b will result in another offset error contribution. This contribution due to power supply variations ΔV is

$$\frac{\partial I_{OE}}{\partial V}\,\Delta V, \tag{28}$$

where $\partial I_{OE}/\partial V$ is the sensitivity coefficient to the power supply voltage found in the datasheet and is typically expressed in $\mathrm{ppm/V}$. Finally, a change in temperature from 23^oC will affect both the mismatch of magnetic properties ϵ_b and the drive electronics, and in turn H_{off}. This contribution due to temperature deviation from 23^oC ΔT is

$$\frac{\partial I_{OE}}{\partial T}\,\Delta T, \tag{29}$$

where $\partial I_{OE}/\partial T$ is the sensitivity coefficient to the temperature found in the datasheet and is typically expressed in $\mathrm{ppm/K}$. We can now get the total relative offset error

$$I_{OE,t} = I_{OE} + \frac{\partial I_{OE}}{\partial B}\,\Delta B + \frac{\partial I_{OE}}{\partial V}\,\Delta V + \frac{\partial I_{OE}}{\partial T}\,\Delta T. \tag{30}$$

To convert this total relative offset error into an absolute error referenced to the primary we multiply with the **appropriate nominal current specification**

$$I_{OE,t}\big|^{p} = I_{OE,t} \cdot 10^{-6} \cdot I_{PN}. \tag{31}$$

To convert it to an absolute error referred to the secondary or output we can plug (31) into (19) to get

$$I_{OE,t}\big|^{o} = I_{OE,t} \cdot 10^{-6} \cdot \frac{I_{PN}}{k}.$$

(32)

The are a few subtleties to note here: Equation (32) gives us the output we can expect to measure from a transducer when there is zero primary current. It is expressed in $\mathrm{Amperes}$ for current output transducers and Volts for voltage output transducers. Consequently, equation (31) is always expressed in $\mathrm{Amperes}$, and it gives us the erroneous primary current we can expect to measure only by looking at the transducer output, when there is truly zero primary current. In practice, equation (31) gives us the error in primary current we can expect due to the offset error by simply measuring the transducer output and multiplying it by the turns ratio N or transfer ratio k.

An Offset stability specification is given in $\mathrm{ppm/month}$ that is very small and we don't include it in the Error budget. If desired, it can be added using the same methodology. We use this specification to make a judgement call about the stability of the offset when we discuss offset compensation/removal.

For voltage output transducers, the electronics can add an additional offset. This effect is included in the offset specification of a voltage output transducer; however, if using a current output transducer with a separate VOM in a DSSIU-V, we need to factor that in. VOM errors will be discussed subsequently.

Total Accuracy and Phase Shift

The Total Accuracy and Phase Shift are error specifications characterizing the frequency response of a transducer that is very complex as we will discuss. Closed-loop fluxgate transducers are specified as low-pass filters responding from DC up to a high-frequency cutoff frequency, referred to as the Bandwidth; this is an overly simplified model.

A typical passive CT has an upper cutoff frequency inversely proportional to the effective winding capacitance multiplied by the $50\,\mathrm{Ohm}$ termination impedance, while its transfer function rolls off at $-20\,\mathrm{dB/decade}$ (for the most part). One would expect that since the high-frequency part of the response of a closed-loop fluxgate is given by a passive CT the response would be similar; this is not true.

As we can see in Figure 6, a typical closed-loop fluxgate sensor comprises three magnetic cores with several windings seeing different termination impedances and additional shielding, creating a very complex electromagnetic problem. There can be several closely spaced poles and zeros after a certain frequency, and as a result the Danisense transducer's Bandwidth does not correspond to a $-3\,\mathrm{dB}$ point as is typical for passive CTs and other Linear Time Invariant Systems. Instead, the bandwidth is typically specified as the $\pm X\,\mathrm{dB}$ frequency point, up to which the transfer function is well behaved. The $\pm$ in the specification implies that the gain can actually be increasing instead of decreasing at the bandwidth frequency. The Bandwidth is often specified as a $\pm 3\,\mathrm{dB}$ frequency point, but sometimes it is a different number, e.g. $\pm 1\,\mathrm{dB}$ in the DS50ID.

The Total Accuracy specifications are given as $\%\,of\,reading + \%\,of\,full\,scale$ at different frequency intervals. If we denote these two numbers at a given frequency interval with $\epsilon_{tot,reading}$ and $\epsilon_{tot,FS}$, then we can calculate at absolute Total Accuracy referred to the primary on that interval as follows

$$\epsilon_{tot,t}\big|^{p} = \epsilon_{tot,reading} \cdot 10^{-2} \cdot I_p + \epsilon_{tot,FS} \cdot 10^{-2} \cdot I_{PN}.$$

(33)

Note that equation (33) gives us the error we can expect in primary current in $\mathrm{Amperes}$ because of the transfer function of the sensor. Because the Total Accuracy is given as $\%\, of\ reading + \%\, of\ full\, scale$, the relative to the primary current Total Accuracy in ppm would need to get calculated from equation (33) as follows

$$\epsilon_{tot,t} = \frac{\epsilon_{tot}|^{p}}{I_p} \cdot 10^6 = \epsilon_{tot,reading} \cdot 10^4 + \epsilon_{tot,FS} \cdot 10^4 \frac{I_{PN}}{I_p}.$$

(34)

We can see from equation (34) that the relative to the primary current Total Accuracy error increases with reduced primary current. As mentioned in the Danisense specifications, the total accuracy at a frequency point within a range can be calculated by linear interpolation between the number of that range and the lower one, unless the frequency point is within the <10Hz range. However, for numerical simplicity the Total Accuracy of the enclosing range can be used instead, recognizing that it would overstate the error.

The Phase Shift specification is given directly in degrees at different frequency intervals. The previous comments about calculation of the Total Accuracy at a specific frequency also apply to the phase shift.

Observing the numbers given in the specifications for the Total Accuracy and the Phase Shift, we can observe that the error and phase increase with increased frequency, and then dramatically at the last interval corresponding to the Bandwidth. This is because the transfer function is approaching a pole or a zero and the gain and phase shift start deviating from the flat response. We can also observe that there tends to be a large increase in error between the 100 Hz and 1 kHz ranges where we expect handover from the fluxgate to the passive CT driving the feedback loop.

Ratio Error (only for Voltage Output Transducers)

In voltage output transducers, a VOM converts the secondary current of a current output transducer into a voltage. This is implemented with a precision burden resistor, and potentially a signal conditioning circuit based on an operational amplifier and additional resistors and capacitors. As we discussed earlier, the transfer ratio of the system is no longer based on a number of turns that is very precise but rather depends on resistors that add additional errors in the transfer ratio. The Ratio error is a relative error denoted by ϵ_k and is specified only for voltage output transducers.

Resistor values are affected by the operating temperature; a change in temperature from $23^o\mathrm{C}$ will affect the gain of the VOM. This contribution due to temperature deviation from $23^o\mathrm{C}$ ΔT is

$$\frac{\partial \epsilon_k}{\partial T}\, \Delta T,$$

(35)

where $\partial\epsilon_k/\partial T$ is the sensitivity coefficient found in the datasheet and is typically expressed in $\mathrm{ppm/K}$. We can now get the total Ratio error

$$\epsilon_{k,t} = \epsilon_k + \frac{\partial \epsilon_k}{\partial T}\,\Delta T.$$

(36)

To convert this total relative Ratio error into an absolute error referenced to the primary we multiply with the primary current

$$\epsilon_{k,t}\big|^{p} = \epsilon_{k,t} \cdot 10^{-6} \cdot I_p.$$

(37)

To convert it to an absolute error referenced to the secondary or output we can plug (37) into (19) to get

$$\epsilon_{k,t}\big|^{o} = \epsilon_{k,t} \cdot 10^{-6} \cdot \frac{I_p}{k}.$$

(38)

Note that as in the Offset error, a Ratio stability specification is given in $\mathrm{ppm/month}$ that is very small and we don't include it in the Error budget. However, if needed, it can added using the same methodology as the effect of temperature.

VOM Errors

When using a current output transducer with a DSSIU-V and a VOM, then the errors of the transducer and the VOM are given separately. We will now discuss the VOM errors in the DSSIU-V datasheet. We will add a VOM superscript in the notation so that we can differentiate from the transducer error when we combine them in a subsequent section.

As in the voltage output transducers, the Ratio error is a relative error denoted by ϵ_k^{VOM}. The error contribution due to temperature deviation from $23^o\mathrm{C}$ ΔT is

$$\frac{\partial \epsilon_k^{VOM}}{\partial T}\,\Delta T,$$

(39)

where $\partial \epsilon_k^{VOM}/\partial T$ is the sensitivity coefficient found in the datasheet and is typically expressed in $\mathrm{ppm/K}$. We can now get the total Ratio error

$$\epsilon_{k,t}^{VOM} = \epsilon_k^{VOM} + \frac{\partial \epsilon_k^{VOM}}{\partial T}\,\Delta T.$$

(40)

To convert this total relative Ratio error into an absolute error referenced to the primary of the current transducer connected to the VOM we multiply with the primary current

$$\epsilon_{k,t}^{VOM}\big|^{p} = \epsilon_{k,t} \cdot 10^{-6} \cdot I_p.$$

(41)

A Ratio stability specification is again given in $\mathrm{ppm/month}$ that is very small and we don't include it in the Error budget.

As with the transducers, a Linearity error is specified for the VOM to capture nonlinearities. It is a relative error denoted by ϵ_L^{VOM} and is not affected by the environment, i.e. $\epsilon_{L,t}^{VOM} = \epsilon_L^{VOM}$. To convert this total relative Ratio error into an absolute error referenced to the primary of the current transducer connected to the VOM we multiply with the primary current

$$\epsilon_{L,t}^{VOM}\big|^{p} = \epsilon_{L,t} \cdot 10^{-6} \cdot I_p.$$

(42)

Finally, an Offset error is specified for the VOM to capture an offset produced by the operational amplifier. It is specified as an absolute error referenced to the output $V_{OE}^{VOM}|^{o}$. The error contribution due to temperature deviation from $23^{o}C$ ΔT is

$$\frac{\partial}{\partial T} V_{OE}^{VOM}|^{o}\, \Delta T,$$

(43)

where $\partial V_{OE}^{VOM}|^{o}/\partial T$ is the sensitivity coefficient found in the datasheet and is typically expressed in $\mu V/K$. We can now get the total VOM Offset absolute error referenced to the output

$$V_{OE,t}^{VOM}\big|^{o} = V_{OE}^{VOM}|^{o} + \frac{\partial}{\partial T} V_{OE}^{VOM}|^{o}\, \Delta T.$$

(44)

We can convert this into an absolute error referenced to the primary of the current-output transducer using equation (19) with the total system gain $k \to N \cdot k^{VOM}$ as follows

$$V_{OE,t}^{VOM}\big|^{p} = N \cdot k^{VOM} \cdot V_{OE,t}^{VOM}\big|^{o} = N \cdot k^{VOM} \cdot \left(V_{OE}^{VOM}|^{o} + \frac{\partial}{\partial T} V_{OE}^{VOM}|^{o}\, \Delta T \right).$$

(45)

Calculating the Error Budget for Current and Voltage Output

Now that we have established a notation and explained the different errors and the effect of the environment, we can proceed to perform calculations for current-output and voltage-output transducers. The current-output transducer with a VOM will be separately analyzed in the next section. We will summarize the analytical calculations and show an example using the DS50ID and the DS50UB-10V. The next section will show an example using the DS50ID with the VOM0100-10V that is effectively the same instrument as the DS50UB-10V.

The goal in each case is to calculate the error budget in the measurement of the primary current ΔI_p. The process is the following

1. We start by setting the operating conditions of interest: primary current I_p, temperature difference from $23^{o}C$, frequency, magnetic field above background ΔB, and deviation from nominal power supply voltage ΔV.
2. We then record the transducer specifications, i.e. nominal current I_{PN}, transfer ratio k, and the all the relevant errors and sensitivity coefficients to environment parameters.
3. We calculate the total relative errors by adding the effect of the environment when applicable, except for the Total Accuracy.
4. We convert into absolute errors referenced to the primary current.
5. We sum all the absolute errors referenced to the primary current to get ΔI_p.

In the following examples, we will assume no environmental effects for numerical simplicity; however, they will be included in the analytical formulas for reference.

Current Output Transducers Example

Step 1: Set the Operating Conditions

We will calculate the error budget of the DS50ID for $10\,\mathrm{A}$ DC at nominal environmental conditions:

$$I_p = 10\ \mathrm{A}$$

$$\Delta T = T - 23^o\mathrm{C} = 0^o\ \mathrm{K}$$

$$\Delta B = 0\ \mathrm{mT}$$

$$\Delta V = 0\ \mathrm{V}$$

(46)

Step 2: Record Transducer Specifications

$$I_{PN} = I_{PN,DC} = 75\ \mathrm{A}$$

$$k = 500$$

$$\epsilon_L = 1\ \mathrm{ppm}$$

$$I_{OE} = 60\ \mathrm{ppm}$$

$$\frac{\partial I_{OE}}{\partial T} = 0.4\ \mathrm{ppm/K}$$

$$\frac{\partial I_{OE}}{\partial B} = 40\ \mathrm{ppm/mT}$$

$$\frac{\partial I_{OE}}{\partial V} = 1.2\ \mathrm{ppm/V}$$

$$\epsilon_{tot,reading} = 0.0002\ \%$$

$$\epsilon_{tot,FS} = 0.00004\ \%$$

(47)

Step 3: Calculate Total Relative Errors

$$\epsilon_{L,t} = \epsilon_L = 1\ \mathrm{ppm}$$

$$I_{OE,t} = I_{OE} + \frac{\partial I_{OE}}{\partial B}\,\Delta B + \frac{\partial I_{OE}}{\partial V}\,\Delta V + \frac{\partial I_{OE}}{\partial T}\,\Delta T = 60\ \mathrm{ppm}$$

(48)

Step 4: Convert Total Relative Errors to Total Absolute Errors Referenced to Primary

$$\epsilon_{L,t}\big|^{p} = \epsilon_{L,t} \cdot 10^{-6} \cdot I_p = 10\ \mu\mathrm{A}$$

$$I_{OE,t}\big|^{p} = I_{OE,t} \cdot 10^{-6} \cdot I_{PN} = 4{,}500\ \mu\mathrm{A}$$

$$\epsilon_{tot,t}\big|^{p} = \epsilon_{tot,reading} \cdot 10^{-2} \cdot I_p + \epsilon_{tot,FS} \cdot 10^{-2} \cdot I_{PN} = 50\ \mu\mathrm{A}$$

(49)

Step 5: Sum the Total Absolute Errors

$$\Delta I_p = \epsilon_{L,t}\big|^p + I_{OE,t}\big|^p + \epsilon_{tot,t}\big|^p = 4{,}560\ \mu\text{A}$$

$$\frac{\Delta I_p}{I_p} = 0.0456\ \%$$

(50)

Voltage Output Transducers Example

Step 1: Set the Operating Conditions

We will calculate the error budget of the DS50UB-10V for $10\ \text{A}$ DC at nominal environmental conditions:

$$I_p = 10\ \text{A}$$

$$\Delta T = T - 23^o\text{C} = 0^\text{o}\ \text{K}$$

$$\Delta B = 0\ \text{mT}$$

$$\Delta V = 0\ \text{V}$$

(51)

Step 2: Record Transducer Specifications

$$I_{PN} = I_{PN,DC} = 50\ \text{A}$$

$$k = 5\ \text{A/V}$$

$$\epsilon_L = 5\ \text{ppm}$$

$$\epsilon_k = 50\ \text{ppm}$$

$$\frac{\partial \epsilon_k}{\partial T} = 3\ \text{ppm/K}$$

$$I_{OE} = 90\ \text{ppm}$$

$$\frac{\partial I_{OE}}{\partial T} = 1\ \text{ppm/K}$$

$$\frac{\partial I_{OE}}{\partial B} = 50\ \text{ppm/mT}$$

$$\frac{\partial I_{OE}}{\partial V} = 4\ \text{ppm/V}$$

$$\epsilon_{tot,reading} = 0.0055\ \%$$

$$\epsilon_{tot,FS} = 0.0001\ \%$$

(52)

Step 3: Calculate Total Relative Errors

$$\epsilon_{L,t} = \epsilon_L = 5\ \text{ppm}$$

$$\epsilon_{k,t} = \epsilon_k + \frac{\partial \epsilon_k}{\partial T} \Delta T = 50 \text{ ppm}$$

$$I_{OE,t} = I_{OE} + \frac{\partial I_{OE}}{\partial B} \Delta B + \frac{\partial I_{OE}}{\partial V} \Delta V + \frac{\partial I_{OE}}{\partial T} \Delta T = 90 \text{ ppm}$$

(53)

<u>Step 4:</u> Convert Total Relative Errors to Total Absolute Errors Referenced to Primary

$$\epsilon_{L,t}\big|^{p} = \epsilon_{L,t} \cdot 10^{-6} \cdot I_p = 50 \text{ µA}$$

$$\epsilon_{k,t}\big|^{p} = \epsilon_{k,t} \cdot 10^{-6} \cdot I_p = 500 \text{ µA}$$

$$I_{OE,t}\big|^{p} = I_{OE,t} \cdot 10^{-6} \cdot I_{PN} = 4{,}500 \text{ µA}$$

$$\epsilon_{tot,t}\big|^{p} = \epsilon_{tot,reading} \cdot 10^{-2} \cdot I_p + \epsilon_{tot,FS} \cdot 10^{-2} \cdot I_{PN} = 600 \text{ µA}$$

(54)

<u>Step 5:</u> Sum the Total Absolute Errors

$$\Delta I_p = \epsilon_{L,t}\big|^{p} + \epsilon_{k,t}\big| + I_{OE,t}\big|^{p} + \epsilon_{tot,t}\big|^{p} = 5{,}650 \text{ µA}$$

$$\frac{\Delta I_p}{I_p} = 0.0565\ \%$$

(55)

Current Output Transducer and VOM Error Budget

Calculating the error budget for a Current Output transducer connected to a VOM requires gymnastics between some absolute errors referenced to the primary current and some to the output of the VOM. We start by writing the output current I_s of the transducer having turns ratio N as a function of the primary current and the applicable total absolute errors referenced to the primary:

$$I_s = \frac{I_p + \epsilon_L|^{p} + I_{OE,t}\big|^{p} + \epsilon_{tot,t}\big|^{p}}{N}.$$

(56)

We then write the input current I_s to the VOM having transfer ratio k^{VOM} as a function of the output voltage V_o and the applicable total absolute errors referenced to the output:

$$I_s = k^{VOM}\left(V_o - \epsilon_{L,t}^{VOM}\big|^{o} - \epsilon_{k,t}^{VOM}\big|^{o} - V_{OE,t}^{VOM}\big|^{o}\right).$$

(57)

The total error for the system is

$$\Delta I_p = N \cdot k^{VOM} \cdot V_o - I_p.$$

(58)

Plugging equations (56) and (57) into (58) yields

$$\Delta I_p = N\cdot k^{VOM}\cdot\left(\frac{I_s}{k^{VOM}} + \epsilon_{L,t}^{VOM}\Big|^{o} + \epsilon_{k,t}^{VOM}\Big|^{o} + V_{OE,t}^{VOM}\Big|^{o}\right) - N\cdot I_s + \epsilon_L|^{p} + I_{OE,t}\Big|^{p} + \epsilon_{tot,t}\Big|^{p}.$$

(59)

Using equation (19) with the total system gain $k \to N\cdot k^{VOM}$ and rearranging terms, equation (59) becomes

$$\Delta I_p = \epsilon_L|^{p} + I_{OE,t}\Big|^{p} + \epsilon_{tot,t}\Big|^{p} + \epsilon_{L,t}^{VOM}\Big|^{p} + \epsilon_{k,t}^{VOM}\Big|^{p} + V_{OE,t}^{VOM}\Big|^{p}.$$

(60)

Example

Step 1: Set the Operating Conditions

We will calculate the error budget of the DS50ID connected to a VOM0100-10 for $10\ \mathrm{A}$ DC at nominal environmental conditions:

$$I_p = 10\ \mathrm{A}$$

$$\Delta T = T - 23^{o}\mathrm{C} = 0^{\mathrm{o}}\ \mathrm{K}$$

$$\Delta B = 0\ \mathrm{mT}$$

$$\Delta V = 0\ \mathrm{V}$$

(61)

Step 2a: Record Transducer Specifications

$$I_{PN} = I_{PN,DC} = 75\ \mathrm{A}$$

$$N = 500$$

$$\epsilon_L = 1\ \mathrm{ppm}$$

$$I_{OE} = 60\ \mathrm{ppm}$$

$$\frac{\partial I_{OE}}{\partial T} = 0.4\ \mathrm{ppm/K}$$

$$\frac{\partial I_{OE}}{\partial B} = 40\ \mathrm{ppm/mT}$$

$$\frac{\partial I_{OE}}{\partial V} = 1.2\ \mathrm{ppm/V}$$

$$\epsilon_{tot,reading} = 0.0002\ \%$$

$$\epsilon_{tot,FS} = 0.00004\ \%$$

(62)

Step 2b: Record VOM Specifications

$$k = \frac{1}{100} = 0.01\ \mathrm{A/V}$$

$$\epsilon_L^{VOM} = 5\ \mathrm{ppm}$$

$$\epsilon_k^{VOM} = 50 \text{ ppm}$$

$$\frac{\partial \epsilon_k^{VOM}}{\partial T} = 3 \text{ ppm/K}$$

$$V_{OE}^{VOM}|^o = 3 \text{ µV}$$

$$\frac{\partial}{\partial T} V_{OE}^{VOM}|^o = 0.2 \text{ µV/K}$$

(63)

Step 3a: Calculate Total Relative Errors for the Transducer

$$\epsilon_{L,t} = \epsilon_L = 1 \text{ ppm}$$

$$I_{OE,t} = I_{OE} + \frac{\partial I_{OE}}{\partial B}\,\Delta B + \frac{\partial I_{OE}}{\partial V}\,\Delta V + \frac{\partial I_{OE}}{\partial T}\,\Delta T = 60 \text{ ppm}$$

(64)

Step 3b: Calculate Total Relative Errors for the VOM

$$\epsilon_{L,t}^{VOM} = \epsilon_L^{VOM} = 5 \text{ ppm}$$

$$\epsilon_{k,t} = \epsilon_k + \frac{\partial \epsilon_k}{\partial T}\,\Delta T = 50 \text{ ppm}$$

(65)

Step 4: Convert Total Relative Errors to Total Absolute Errors Referenced to Primary

$$\epsilon_{L,t}\big|^p = \epsilon_{L,t} \cdot 10^{-6} \cdot I_p = 10 \text{ µA}$$

$$I_{OE,t}\big|^p = I_{OE,t} \cdot 10^{-6} \cdot I_{PN} = 4{,}500 \text{ µA}$$

$$\epsilon_{tot,t}\big|^p = \epsilon_{tot,reading} \cdot 10^{-2} \cdot I_p + \epsilon_{tot,FS} \cdot 10^{-2} \cdot I_{PN} = 50 \text{ µA}$$

$$\epsilon_{L,t}^{VOM}\big|^p = \epsilon_{L,t}^{VOM} \cdot 10^{-6} \cdot I_p = 50 \text{ µA}$$

$$\epsilon_{k,t}^{VOM}\big|^p = \epsilon_{k,t}^{VOM} \cdot 10^{-6} \cdot I_p = 500 \text{ µA}$$

$$V_{OE,t}^{VOM}\big|^p = N \cdot k \cdot \left(V_{OE}^{VOM}|^o + \frac{\partial}{\partial T} V_{OE}^{VOM}|^o \cdot \Delta T\right) = 15 \text{ µA}$$

(66)

Step 5: Sum the Total Absolute Errors

$$\Delta I_p = \epsilon_{L,t}\big|^p + I_{OE,t}\big|^p + \epsilon_{tot,t}\big|^p + \epsilon_{L,t}^{VOM}\big|^p + \epsilon_{k,t}^{VOM}\big|^p + V_{OE,t}^{VOM}\big|^p = 5{,}125 \text{ µA}$$

$$\frac{\Delta I_p}{I_p} = 0.05125\ \%$$

(67)

Offset Error Compensation

As must be evident from the above, the Offset is the largest error dominating the error budget at low currents. It is also the only error that can be systematically removed given certain conditions. External magnetic field aside, the Offset error rises primarily from the mismatch in magnetic properties between the two cores that in turn is affected by the magnetic history of the cores.

Although our simple B-H loop did not capture it, the magnetic material of the core has hysteresis. When we power on and off a transducer the phase of the internal oscillator driving the excitation signal is unpredictable. Therefore, there is an unpredictable sequence of amplitude excitations as the power supply voltage ramps up and down. This unpredictable power-on and power-off sequence coupled with the hysteresis of the material leaves the cores in a slightly different magnetic state after each power cycle, and therefore with slightly different magnetic properties. Because of these slightly different magnetic properties after each power cycle, the transducer offset will change between power cycles because of the term ϵ_b in equation (13).

The Offset current in the Danisense specifications captures this effect of an offset current because of the magnetic history of the cores in the transducer. However, as shown in the Offset stability over time specification, once a transducer has been powered on, has thermally stabilized, and the temperature and background magnetic field don't change, then the offset is extremely stable (sub-ppm/month).

It follows that we can remove the Offset error as follows: Once the transducer is powered on and has thermally stabilized, we measure the output X_{off} with no primary current. Then we start taking measurements of the output of the transducer X_o with finite primary current I_p. We can then calculate the primary current as

$$I_p = k\left(X_o - X_{off}\right).$$

(68)

This removes the offset. Alternatively, if using a power analyzer, we can use the Zero functionality when the primary current is zero. The power analyzer will then be internally performing the computation of equation (68) when taking data.

Note that for this procedure to be accurate, the temperature and the background magnetic field need to remain stable as when the offset was measured or the power analyzer was zeroed and the transducer must not get power cycled. Also note that if the internal protection (ASPC) circuit is triggered this will effectively power cycle the transducer and the offset must be re-measured.

A judgement call must then be made about how frequently to re-measure the offset (other than after a power cycle). Even if the transducer is not power cycled and the temperature and background magnetic field are stable, there will be long-term drifts, as expressed by the Offset stability over time specification. Over the course of a month the offset is not expected to drift substantially; however, over the course of a year the drift may be unacceptable.

Fluxgate Excitation Frequency & Internal Noise

The excitation frequency is listed on the Danisense specifications and ranges from a little over $30\,\mathrm{kHz}$ for the low-current transducers to less than $10\,\mathrm{kHz}$ for the very high current transducers. As shown in Figure 6, the excitation windings are antisymmetric with respect to the feedback windings between the two cores and therefore the excitation signal should cancel in the output. However, because of various imperfections, as well as the presence of multiple windings, the excitation signal does break through to the output signal of the transducer.

High frequency broadband current measurements show noise at the excitation frequency and its (primarily odd) harmonics. This fact is captured in the noise specification of the transducers. Danisense specifies the transducer-generated noise at the output signal as RMS and peak-to-peak in ppm relative to the nominal current at different frequency intervals, like the total accuracy and phase specifications. Observing the numbers, we can see a gradual noise increase with increased frequency corresponding to Johnsons-Nyquist Noise $\overline{V_n^2} = 4k_B TR\Delta f$ [4], until the frequency interval includes the excitation frequency. Then noise rises sharply due to the excitation frequency breakthrough.

We did not add the noise as an additional error in the error budget to avoid added complexity that is unnecessary for most users. If the noise needs to be captured as an additional error, then it can be considered an additional total relative error $\epsilon_{n,t}$ and converted to an absolute error $\epsilon_{n,t}\big|^p$ referred to the primary current by multiplying with the nominal AC current

$$\epsilon_{n,t}\big|^p = \epsilon_{n,t} \cdot 10^{-6} \cdot I_{PN\ AC}.$$

(69)

This absolute error can be added to the total error using the same methodology previously discussed.

Noise Pickup & Measurement Instrument Considerations

For customers working in environments with significant electromagnetic noise, such as around high-power motor drives, and using long cables, the recommendation is always to use a current output transducer. If a voltage output is desired, then the recommendation is to use a current transducer with a VOM in a DSSIU-V very close to the measurement instrument. In this section we will use a simplified circuit model to explain this recommendation. Electromagnetic Interference and mitigation are a very broad topic outside the scope of this note; the reader is referred to [5].

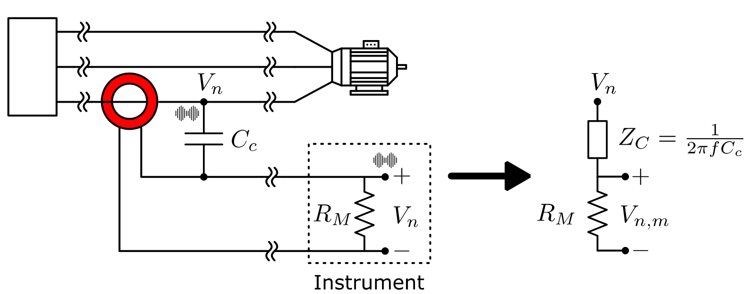

Figure 9:Simplified schematic for explaining noise pickup.

Figure 9 shows a simplified schematic to describe the effect of the measurement instrument on the sensitivity to noise. A DCCT is measuring one conductor in a 3-phase motor drive system. The output of the DCCT is connected with a long cable to a measurement instrument having input impedance R_M. The high-power cable is capacitively coupled to the cable used for the measurement. The impedance of the capacitive coupling C_C at some frequency f is

$$Z_C = \frac{1}{2\,\pi\,f\,C_C}. \tag{70}$$

Z_C forms an impedance divider with the input impedance R_M of the instrument. Let's assume there is a voltage noise signal at frequency f and amplitude V_n on the high-power line. This signal would propagate through the impedance divider and result in measured noise at the instrument of amplitude

$$V_{n,m} = \frac{R_M}{R_M+Z_C}V_n. \tag{71}$$

If we have a noise signal at $f = 10\,\mathrm{kHz}$ and $C_C = 100\,\mathrm{pF}$ that is reasonable for the example application, then $Z_C = 160\,\mathrm{k\Omega}$. The typical input impedance of the voltage channel of a typical DMM is $R_M^V = 10\,\mathrm{M\Omega}$. Plugging these numbers into equation (71) yields

$$V_{n,m}^V = \frac{10\,\mathrm{M\Omega}}{10\,\mathrm{M\Omega}+160\,\mathrm{k\Omega}}V_n \approx V_n. \tag{72}$$

The typical input impedance of the current channel of a typical DMM is very low; let's assume $R_M^I = 1\,\Omega$. Plugging these numbers into equation (71) yields

$$V_{n,m}^I = \frac{1\,\Omega}{1\,\Omega+160\,\mathrm{k\Omega}}V_n \approx 0. \tag{73}$$

From the preceding, we can see that a voltage-output transducer terminated into high-impedance will result in the full amplitude of the noise transferred to the measurement. In comparison, a current-output transducer terminated into low-impedance will result in the noise amplitude being highly attenuated. In the case of a current-output transducer with a VOM, the VOM also presents a low input impedance. The output of the VOM will be terminated into high-impedance to make a voltage measurement; however, if the DSSIU-V and the measurement instrument are far away from the noise source then the capacitive coupling to the cable from the DSSIU-V to the measurement instrument will be insignificant.

Summary: Choosing the Right Output for your Application

As a summary of this Application Note, choosing the appropriate transducer output for most applications will come down to these factors:

- Measurement Instrument: If using an oscilloscope, then a voltage output system is needed. If using a DMM or Power Analyzer, the question then becomes about balancing errors between the transducer and instrument. Voltage inputs of DMMs and Power Analyzers tend to have much better accuracy compared to Current inputs; however, voltage output transducers tend to have larger errors than current output transducers.

One must examine the instrument and transducer together. If a voltage-output system is preferred, then a voltage-output transducer and a current-output with a VOM will have very similar error specifications.

- Installation: A voltage-output transducer will require two cables to be routed, one from the transducer to the DSSIU and the other from the transducer to the measurement instrument. As we saw, a current output transducer will be more immune to noise compared to a voltage-output transducer, whether there is a VOM or not on the DSSIU. It follows that if we need to use a voltage-output system in a noisy environment, then a current-output transducer with a VOM will be more immune to noise than a voltage-output transducer.
- Flexibility: A DSSIU without VOMs can be used with any transducer model, whether current-output or voltage-output. However, a VOM will lock a DSSIU-V channel to a very specific current-output transducer model. Note the voltage-output transducers are incompatible to DSSIU-V because the internal burden resistors of the VOM and transducer would get connected in parallel.

Attachments

- Mathematica Notebook for calculating the Error Budget.

References

[1] Sharp, J. B. (1962). The induction type beam monitor for the PS: Hereward transformer (No. MPS-Int-CO-62-15).

[2] Unser, K. B. (1992, April). The parametric current transformer, a beam current monitor developed for LEP. In AIP Conference Proceedings (Vol. 252, No. 1, pp. 266-275). American Institute of Physics.

[3] Musmann, G. (Ed.). (2010). Fluxgate magnetometers for space research. BoD–Books on Demand.

[4] https://en.wikipedia.org/wiki/Johnson%E2%80%93Nyquist_noise

[5] Ott, H. W. (2011). Electromagnetic compatibility engineering. John Wiley & Sons.

Out[ ]=

GMW Associates

Application Note: A Guide to Error Analysis in Danisense DCCT Systems and Considerations for Output Selection

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This Mathematica Notebook accompanies the GMW Application Note: A Guide to Error Analysis in Danisense DCCT Systems and Considerations for Output Selection. It provides numerical examples of calculating error budgets.

DISCLAIMER

Current-Output Transducer Example

Step 1: Set the Operating Conditions

In[ ]:=
```
Ip = 10.; (* Primary Current in A for DC or Arms for AC *)
temp = 23.0 ;(* Ambient Temperature in DegC *)
DT = temp - 23.0;(* Temperature Deviation from 23 DegC *)
DB = 0.0; (* Magnetic Field in mT above Earth's Field *)
DV = 0.0; (* Power Supply Voltage Deviation from Nominal in V *)
```

Step 2: Record Transducer Specifications

In[ ]:=
```
IPN = 75.0; (* Nominal Current in A for DC or Arms for AC *)
k = 500.0 ;(* Transfer Ratio *)
```

Linearity Error

```
In[∘]:= ϵL = 1.0; (* Linearity Error in ppm *)
```

Offset Error

```
In[∘]:= IOE = 60.0; (* Offset Current in ppm *)
DIOEDT = 0.4; (* Offset Temperature Coefficient in ppm/K *)
DIOEDB = 40.0; (* Offset change with external magnetic field in ppm/mT *)
DIODV = 1.2; (* Offset change with power supply voltage changes in ppm/V*)
```

Total Accuracy

```
In[∘]:= ϵTotR = 0.0002; (* Total Accuracy % of Reading *)
ϵTotFS = 0.00004; (* Total Accuracy % of Full Scale *)
```

Step 3: Calculate Total Relative Errors

```
In[∘]:= (* Calculate & Print Total Linearity Error *)
ϵLt = ϵL;
Print["Tot. Linearity Error: " <> ToString[ϵLt] <> " ppm"]

(* Calculate & Print Total Offset Error *)
IOEt = IOE + DIOEDT * DT + DIOEDB * DB + DIODV * DV;
Print["Tot. Offset Error: " <> ToString[IOEt] <> " ppm"]
```

```
Tot. Linearity Error: 1. ppm

Tot. Offset Error: 60. ppm
```

Step 4: Convert to Total Absolute Errors Referenced to the Primary

```
In[∘]:= (* Calculate & Print Absolute Linearity Error *)
ϵLtTot = ϵLt * Ip;
Print["Linearity Error @ Ip: " <> ToString[ϵLtTot] <> " uA or uArms"]

(* Calculate & Print Absolute Offset Error *)
IOEtTot = IOEt * IPN;
Print["Offset Error: " <> ToString[IOEtTot] <> " uA or uArms"]

(* Calculate & Print Absolute Total Accuracy Error *)
ϵTotTot = ϵTotR * Ip * 10^4 + ϵTotFS * IPN * 10^4;
Print["Total Accuracy Error @ Ip: " <> ToString[ϵTotTot] <> " uA or uArms"]
```

```
Linearity Error @ Ip: 10. uA or uArms

Offset Error: 4500. uA or uArms

Total Accuracy Error @ Ip: 50. uA or uArms
```

Step 5: Add to Get DIp

In[ ]:=
```
DIp = ϵLtTot + IOEtTot + ϵTotTot;
Print["DIp @ Ip: " <> ToString[DIp] <> " uA or uArms"]
Print["DIp/Ip: " <> ToString[DIp / Ip * 10^-4] <> " %"]
```

```
DIp @ Ip: 4560. uA or uArms
DIp/Ip: 0.0456 %
```

Voltage-Output Transducer Example

Step 1: Set the Operating Conditions

In[ ]:=
```
Ip = 10.0; (* Primary Current in A for DC or Arms for AC *)
temp = 23.0 ; (* Ambient Temperature in DegC *)
DT = temp - 23.0; (* Temperature Deviation from 23 DegC *)
DB = 0.0; (* Magnetic Field in mT above Earth's Field *)
DV = 0.0; (* Power Supply Voltage Deviation from Nominal in V *)
```

Step 2: Record Transducer Specifications

In[ ]:=
```
IPN = 50.0; (* Nominal Current in A for DC or Arms for AC *)
k = 5.0 ; (* Transfer Ratio *)
```

Linearity Error

In[ ]:=
```
ϵL = 5.0; (* Linearity Error in ppm *)
```

Ratio Error

In[ ]:=
```
ϵk = 50.0; (* Ratio Error in ppm *)
DϵkDT = 3.0; (* Ratio Error Temperature Coefficient in ppm/K *)
```

Offset Error

In[ ]:=
```
IOE = 90.0; (* Offset Current in ppm *)
DIOEDT = 1.0; (* Offset Temperature Coefficient in ppm/K *)
DIOEDB = 50.0; (* Offset change with external magnetic field in ppm/mT *)
DIODV = 4.0; (* Offset change with power supply voltage changes in ppm/V*)
```

Total Accuracy

In[ ]:=
```
ϵTotR = 0.0055; (* Total Accuracy % of Reading *)
ϵTotFS = 0.0001; (* Total Accuracy % of Full Scale *)
```

Step 3: Calculate Total Relative Errors

```
In[*]:= (* Calculate & Print Total Linearity Error *)
ϵLt = ϵL;
Print["Tot. Linearity Error: " <> ToString[ϵLt] <> " ppm"]

(* Calculate & Print Total Rario Error *)
ϵkt = ϵk + DϵkDT * DT;
Print["Tot. Ratio Error: " <> ToString[ϵkt] <> " ppm"]

(* Calculate & Print Total Offset Error *)
IOEt = IOE + DIOEDT * DT + DIOEDB * DB + DIODV * DV;
Print["Tot. Offset Error: " <> ToString[IOEt] <> " ppm"]

Tot. Linearity Error: 5. ppm

Tot. Ratio Error: 50. ppm

Tot. Offset Error: 90. ppm
```

Step 4: Convert to Total Absolute Errors Referenced to the Primary

```
In[*]:= (* Calculate & Print Absolute Linearity Error *)
ϵLtTot = ϵLt * Ip;
Print["Linearity Error @ Ip: " <> ToString[ϵLtTot] <> " uA or uArms"]

(* Calculate & Print Absolute Ratio Error *)
ϵktTot = ϵkt * Ip;
Print["Ratio Error @ Ip: " <> ToString[ϵktTot] <> " uA or uArms"]

(* Calculate & Print Absolute Offset Error *)
IOEtTot = IOEt * IPN;
Print["Offset Error: " <> ToString[IOEtTot] <> " uA or uArms"]

(* Calculate & Print Absolute Total Accuracy Error *)
ϵTotTot = ϵTotR * Ip * 10^4 + ϵTotFS * IPN * 10^4;
Print["Total Accuracy Error @ Ip: " <> ToString[ϵTotTot] <> " uA or uArms"]

Linearity Error @ Ip: 50. uA or uArms

Ratio Error @ Ip: 500. uA or uArms

Offset Error: 4500. uA or uArms

Total Accuracy Error @ Ip: 600. uA or uArms
```

Step 5: Add to Get DIp

```
In[ ]:= DIp = ϵLtTot + ϵktTot + IOEtTot + ϵTotTot;
Print["DIp @ Ip:  " <> ToString[DIp] <> " uA or uArms"]
Print["DIp/Ip:  " <> ToString[DIp / Ip * 10^-4] <> " %"]
```

```
DIp @ Ip: 5650. uA or uArms

DIp/Ip: 0.0565 %
```

Current-Output & VOM Transducer Example

Step 1: Set the Operating Conditions

```
In[ ]:= Ip = 10.0; (* Primary Current in A for DC or Arms for AC *)
temp = 23.0 ; (* Ambient Temperature in DegC *)
DT = temp - 23.0; (* Temperature Deviation from 23 DegC *)
DB = 0.0; (* Magnetic Field in mT above Earth's Field *)
DV = 0.0; (* Power Supply Voltage Deviation from Nominal in V *)
```

Step 2a: Record Transducer Specifications

```
In[ ]:= IPN = 75.0; (* Nominal Current in A for DC or Arms for AC *)
n = 500.0 ; (* Transfer Ratio *)
```

Linearity Error

```
In[ ]:= ϵL = 1.0; (* Linearity Error in ppm *)
```

Offset Error

```
In[ ]:= IOE = 60.0; (* Offset Current in ppm *)
DIOEDT = 0.4; (* Offset Temperature Coefficient in ppm/K *)
DIOEDB = 40.0; (* Offset change with external magnetic field in ppm/mT *)
DIODV = 1.2; (* Offset change with power supply voltage changes in ppm/V*)
```

Total Accuracy

```
In[ ]:= ϵTotR = 0.0002; (* Total Accuracy % of Reading *)
ϵTotFS = 0.00004; (* Total Accuracy % of Full Scale *)
```

Step 2b: Record VOM Specifications

```
In[•]:= k = 1.0 / 100.0
Out[•]=
0.01
```

Linearity Error

```
In[•]:= ϵLVOM = 5.0; (* Linearity Error in ppm *)
```

Ratio Error

```
In[•]:= ϵkVOM = 50.0; (* Ratio Error in ppm *)
DϵkVOMDT = 3.0; (* Ratio Error Temperature Coefficient in ppm/K *)
```

Offset Error

```
In[•]:= VOEVOMtot = 3.0; (* Offset Current in uV *)
DVOEVOMtotDT = 0.2; (* Offset Temperature Coefficient in uV/K *)
```

Step 3a: Calculate Total Relative Errors for DCCT

```
In[•]:= (* Calculate & Print Total Linearity Error *)
ϵLt = ϵL;
Print["CT Tot. Linearity Error: " <> ToString[ϵLt] <> " ppm"]

(* Calculate & Print Total Offset Error *)
IOEt = IOE + DIOEDT * DT + DIOEDB * DB + DIODV * DV;
Print["CT Tot. Offset Error: " <> ToString[IOEt] <> " ppm"]

CT Tot. Linearity Error: 1. ppm

CT Tot. Offset Error: 60. ppm
```

Step 3b: Calculate Total Relative Errors for VOM

```
In[•]:= (* Calculate & Print Total Linearity Error *)
ϵLVOMt = ϵLVOM;
Print["VOM Tot. Linearity Error: " <> ToString[ϵLVOMt] <> " ppm"]

(* Calculate & Print Total Rario Error *)
ϵkVOMt = ϵkVOM + DϵkVOMDT * DT;
Print["VOM Tot. Ratio Error: " <> ToString[ϵkVOMt] <> " ppm"]

VOM Tot. Linearity Error: 5. ppm

VOM Tot. Ratio Error: 50. ppm
```

Step 4: Convert to Total Absolute Errors Referenced to the Primary

In[∘]:=
```
(* Calculate & Print Absolute Linearity Error *)
ϵLtTot = ϵLt * Ip;
Print["CT Linearity Error @ Ip: " <> ToString[ϵLtTot] <> " uA or uArms"]

(* Calculate & Print Absolute Offset Error *)
IOEtTot = IOEt * IPN;
Print["CT Offset Error: " <> ToString[IOEtTot] <> " uA or uArms"]

(* Calculate & Print Absolute Total Accuracy Error *)
ϵTotTot = ϵTotR * Ip * 10^4 + ϵTotFS * IPN * 10^4;
Print["CT Total Accuracy Error @ Ip: " <> ToString[ϵTotTot] <> " uA or uArms"]

(* Calculate & Print Absolute Linearity Error *)
ϵLVOMtTot = ϵLVOMt * Ip;
Print["VOM Linearity Error @ Ip: " <> ToString[ϵLVOMtTot] <> " uA or uArms"]

(* Calculate & Print Absolute Ratio Error *)
ϵkVOMtTot = ϵkVOMt * Ip;
Print["VOM Ratio Error @ Ip: " <> ToString[ϵkVOMtTot] <> " uA or uArms"]

(* Calculate & Print Total Offset Error *)
VOEVOMtott = k * n (VOEVOMtot + DVOEVOMtotDT * DT);
Print["VOM Tot. Offset Error: " <> ToString[VOEVOMtott] <> " uA or uArms"]
```

```
CT Linearity Error @ Ip: 10. uA or uArms

CT Offset Error: 4500. uA or uArms

CT Total Accuracy Error @ Ip: 50. uA or uArms

VOM Linearity Error @ Ip: 50. uA or uArms

VOM Ratio Error @ Ip: 500. uA or uArms

VOM Tot. Offset Error: 15. uA or uArms
```

Step 5: Add to Get DIp

In[∘]:=
```
DIp = ϵLtTot + IOEtTot + ϵTotTot + ϵLVOMtTot + ϵkVOMtTot + VOEVOMtott;
Print["DIp @ Ip: " <> ToString[DIp] <> " uA or uArms"]
Print["DIp/Ip: " <> ToString[DIp / Ip * 10^-4] <> " %"]
```

```
DIp @ Ip: 5125. uA or uArms

DIp/Ip: 0.05125 %
```